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# Optimising route choices for the travelling and charging of battery electric vehicles by considering multiple objectives

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## ABSTRACT

A battery electric vehicle (BEV) reduces greenhouse gas emissions and energy consumption to a greater extent than a conventional internal combustion engine vehicle. However, limited driving range, insufficient charging infrastructure, potentially long charging time and high monetary cost affect the route choices of BEV drivers. To provide BEV drivers with decision-making support for travelling and charging, a multi-objective optimisation model is built to optimise route choices for multiple BEVs. Three objective functions are proposed to minimise total travelling cost components, including travel times, energy consumption and charging costs. The fuzzy programming approach and fuzzy preference relations are introduced to transform the three objective functions into a single objective function that comprehensively considers the three optimisation objectives. A genetic algorithm is then designed to solve the model. In addition, a numerical example is presented to demonstrate the proposed model and solution algorithm. Four weighting conditions for BEV drivers based on different trade-offs among the objectives are considered in the numerical example. Results of the numerical example verify the feasibility and effectiveness of the proposed model and algorithm.

*Keywords:*

Battery electric vehicle

Charging request

Route choice

Multi-objective model

## 1. Introduction

The dependence of human society on petroleum has contributed to the development of serious environmental and energy problems. In China, the energy consumption of the transportation sector accounts for 20% of the total energy consumption of the country (Wang et al., 2014) and is responsible for 8% of the total greenhouse gas (GHG) emissions nationwide (Hao et al., 2015). Given the public concern on climate change and advances in battery technologies, electric vehicles (EVs) have enjoyed

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increasing use in recent years because they are frequently considered better options in terms of GHG emissions and energy consumption than internal combustion engine vehicles (ICEVs) (Onat et al., 2015). In particular, battery EVs (BEVs) have been considered the most promising means of travel towards reducing the GHG emissions of the transportation sector given that they produce zero tailpipe emissions during operation (Diao et al., 2016). However, several disadvantages remain in using BEVs. Firstly, the driving range of BEVs is shorter than that of ICEVs because of the limited capacity of batteries in the former. Secondly, charging infrastructure in regional or metropolitan road networks remains insufficient, thereby causing trouble for drivers who need to recharge their BEVs. Finally, charging a BEV is relatively time consuming, and thus, significantly increases the travel times of BEV drivers. Therefore, limited driving range, insufficient charging infrastructure and long charging time increase driver range anxiety, i.e. the fear of depleting battery energy en route (Pearre et al., 2011). These disadvantages negatively affect the adoption of BEVs. To alleviate these disadvantages, information services on charging stations have been developed in China. These services provide information regarding the locations and availability of charging stations through the Internet, thereby allowing drivers to receive information in mobile devices, such as smartphones. When the battery energy of a BEV is insufficient to reach a particular destination, the driver must choose optimal routes that consider both travelling and charging from his/her departure point to his/her destination based on received information regarding charging stations. Therefore, special attention must be given to the route choice problem in BEVs.

Conventional ICEVs are required to be filled with gas as necessary. Unlike ICEVs, BEVs frequently need charging during trips. Therefore, existing route choice methods for ICEVs cannot be applied to BEVs because the limited driving range and charging behaviour of BEVs are not considered in such strategies. In actual BEV operation, limited driving range and charging behaviour significantly influence the route choice of drivers. Several studies on route choice for BEVs have considered single objectives that aim to minimise energy consumption (Eisner et al., 2011; Andreas et al., 2010), total travel distance (Erdoğan et al., 2012) or charging time (Said et al., 2013). Moreover, several works have focused on building optimal routing models by simultaneously considering multiple objectives. Yang et al. (2013) proposed an optimisation model to determine the optimal charging route. The travel time from the point of origin to a charging station, queuing time for charging and charging time were considered as the objectives of their model. However, charging cost was disregarded. Alizadeh et al. (2015) developed an extended graph method for choosing a charging route that considered travel time and charging cost. The feasibility of their approach was verified via a numerical example. Sweda and Klabjan (2012) proposed a dynamic programming method to find a minimum-cost route when BEVs must charge en route. This strategy aimed to simultaneously minimise charging cost and energy consumption. Kobayashi et al. (2011) presented a route search method that considered route distance, travelling time and charging time. Limited driving range and charging station locations were considered the main constraints. However, these studies have not explored the situation that involves multiple BEVs. The route choices of multiple BEVs interact with one another because of the insufficient number of charging stations.

Few works have analysed the route choice problem in multiple BEVs. He et al. (2014), Jiang and Xie (2014) and Jiang et al. (2014) proposed optimisation models that involve network equilibrium based on network equilibrium theory. For example, He et al. (2014) formulated three mathematical models to describe network equilibrium flow distributions on a road network by considering the flow dependency of energy consumption and charging time. These models aimed to minimise travel time or

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costs. Moreover, several optimal routing and charging models have been proposed to alleviate the negative effects of large-scale charging behaviour on a power distribution network (Putrus et al., 2009; Fernández et al., 2011; Qian et al., 2011; Guo et al., 2011).

Although several works have investigated the route choice problem in BEVs, only few studies have given attention to optimal route choices for travelling and charging multiple BEVs that consider multiple objectives. In general, BEV drivers are required to spend considerable time and cost to charge their vehicles. The characteristic energy consumption of BEVs differs from that of conventional ICEVs. Therefore, comprehensively considering travel time, energy consumption and charging cost is important to successfully address the route choice problem in BEVs.

The approach proposed in this study may be used by charging station operators to provide BEV drivers with optimal routes for travelling and charging. This method may also be utilised by city planners to design public charging infrastructure that will fulfil the needs of BEV drivers.

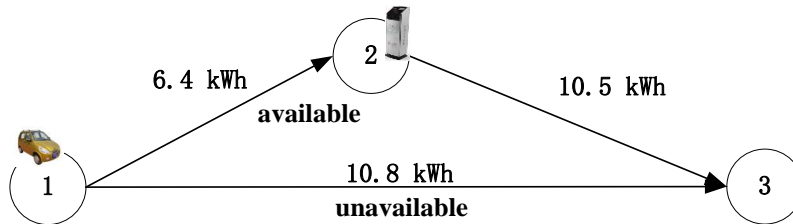
The contributions of this study are as follows. Firstly, a multi-objective optimisation model is built to explore the optimal routes for travelling and charging multiple BEVs. The three objectives are to minimise the total travelling cost components, namely, travel times, energy consumption and charging costs. Secondly, driving time, queuing time and charging time are considered in calculating travel time. The relation between energy consumption per kilometre (ECPK) and driving speed is explored based on BEV operation data. Charging cost includes electricity cost, service cost and parking fee. Lastly, the fuzzy programming approach (FPA) and fuzzy preference relations (FPR) are introduced to transform the proposed multi-objective optimisation model into a model with a single objective function that comprehensively aims to optimise the total travelling cost components. The trade-off among the different travelling cost components for BEV drivers is considered in the model. A genetic algorithm (GA) is designed to address the model, and a numerical example is presented to demonstrate the proposed model and solution algorithm. Moreover, compared to the existing formulations in previous literature, the proposed model simultaneously optimises three objectives with different magnitudes for multiple BEVs, and further considers the interactions of different BEVs. The formulations of the objectives in the model are based on the total travelling cost components of all BEVs that simultaneously make charging requests, and the travel time demand of the individual driver is also considered in the constraints of the model.

The remaining portions of this paper are organised as follows. Section 2 constructs the optimal route choice model with multiple objectives for multiple BEVs. Section 3 discusses the methods for model transformation and introduces the designed solution. Section 4 presents a numerical example to demonstrate and verify the proposed model and solution methods. Lastly, Section 5 provides the conclusions and directions for future research.

## **2. Multi-objective optimisation model for route choices of multiple BEVs**

The proposed model is oriented to address the route choice problem in BEVs. The routes based on driver willingness to deviate from the shortest path to access destinations may not be available for BEVs because of their limited driving range. The distance of an available route must be within the driving range of a BEV, which lacks a charging station along the route. Otherwise, the route is regarded unavailable unless at least one charging station is present along the route and the BEV can be charged to avoid energy depletion before reaching its destination. For example, Fig. 1 illustrates the available and unavailable routes for BEVs. The origin–destination (O–D) pair

1–3 is connected by two routes, namely, 1–3 and 1–2–3. A charging station is located at node 2. We assume that the nominal capacity of a battery is 24 kWh, and the initial energy is 9.6 kWh. The energy consumed to traverse a route depends on the distance and traffic congestion of the route (Bigazzi et al., 2012). We also assume that the energy consumed for traversing routes 1–2, 2–3 and 1–3 are 6.4, 10.5 and 10.8 kWh, respectively (Fig. 1). Then, route 1–3 is easily determined as unavailable because 9.6 kWh is less than 10.8 kWh. Along route 1–2–3, the BEV can reach node 2 to recharge its battery because 9.6 kWh is higher than 6.4 kWh, and then arrive at node 3, which is its destination.



**Fig. 1.** Toy road network with three nodes.

The main objective of the proposed model is to provide decision-making support for BEV drivers in determining optimal routes from departure points to destinations with optimal charging stations along the routes to charge BEVs. To avoid energy depletion and successfully reach their destinations, BEV drivers must choose available routes, which may result in additional travel time and monetary cost compared with their intended routes. These additional costs, which are attributed to charging behaviour, increase driving time and energy consumption because of extra detour distance and different traffic conditions. Therefore, the objective of BEV drivers is to find optimal available routes and charging stations along these routes to minimise travel times, energy consumption and charging costs.

On the basis of BEV operation state, battery state, charging station operational status and driver demands, an optimal route choice model is built in this study by considering three optimisation objectives, i.e. minimising travel time, energy consumption and charging costs.

### 2.1. Basic assumptions

To facilitate model construction, several assumptions are made as follows.

*Assumption 1:* We consider range anxiety and assume that when the locations of departure points, charging stations and destinations are determined, the routes with minimum energy consumption from departure points to charging stations and from charging stations to destinations are chosen.

*Assumption 2:* We assume that BEV drivers make charging requests and charge BEVs only once between departure points and destinations because trips with more than one charging are generally uncommon (Sun et al., 2016). Moreover, for trips with more than one charging, the model can be used repeatedly by setting dummy destinations to address the problem. The dummy destination from the previous charging will be set as the departure point for the next charging.

*Assumption 3:* To reduce model complexity, we assume that all BEVs have batteries with the same nominal capacity.

*Assumption 4:* To reduce model complexity, we assume that chargers in the same charging station

provide the same charging power, and only one charger can be used by one BEV at a particular time.

## 2.2. Objective functions

For model formulation, we assume the existence of  $m$  target BEVs and  $n$  charging stations in a road network ( $i=1, \dots, m; j=1, \dots, n$ ). The route between the departure point and the destination consists of two sub-routes, namely, the route from the departure point to a charging station and the route from the charging station to the destination. For distinction, BEVs that simultaneously make charging requests, i.e. the research targets, are defined as the target BEVs.

The decision variable of the model is the binary variable  $x_{ij}$ , which is equal to 1 if BEV  $i$  chooses charging station  $j$  to charge its battery; otherwise, this variable is 0.

### 2.2.1. Minimising travel times

During the trip, an important factor that influences route and charging station choice is travel time, which includes queuing, charging and driving times. Given the preceding assumption, the objective function for minimising travel times can be expressed as follows:

$$\min T = \min \sum_{i=1}^m \sum_{j=1}^n (t_{ij}^q + t_{ij}^c + t_{ij}^{os} + t_{ij}^{sd}) x_{ij}, \quad (1)$$

Where

$$t_{ij}^q = \begin{cases} \frac{\bar{e} - y_{ijk}^{(r^n)} + (INT(\frac{z_{ij}}{h_j}) - 1)\Delta e}{p_j} & z_{ij} \geq h_j, \\ 0 & z_{ij} < h_j \end{cases}, \quad (2)$$

$$t_{ij}^c = \frac{e_{ij}^{\bar{s}} - e_{ij}^s}{p_j}, \quad (3)$$

$$t_{ij}^{os} = \sum_{a \in A_j^{os}} t_a, \quad (4)$$

$$t_{ij}^{sd} = \sum_{a \in A_j^{sd}} t_a. \quad (5)$$

Objective (1) minimises the total travel times  $T$  of all target BEVs, which involves a zero–one integer programming, where  $t_{ij}^q$  is the queuing time of BEV  $i$  in charging station  $j$ ;  $t_{ij}^c$  is the charging time of target BEV  $i$  in charging station  $j$ ; and  $t_{ij}^{os}$  and  $t_{ij}^{sd}$  are the driving times of target BEV  $i$  operating from its departure point to charging station  $j$  and from charging station  $j$  to its destination, respectively.

Eq. (2) presents the queuing time of target BEV  $i$  in charging station  $j$ , which is a piecewise function. When a BEV reaches a charging station and no charger is free, the driver incurs queuing delay. Otherwise, the BEV is immediately charged using a free charger. Let  $z_{ij}$  denote the number of BEVs in

charging station  $j$  when target BEV  $i$  reaches this charging station. To determine queuing time,  $z_{ij}$  must be estimated.  $z_{ij}$  is equal to the greatest value of zero and  $z_j^o + z_{ij}^s + z_{ij}^d$  where  $z_j^o$  is the number of BEVs in charging station  $j$  when the target BEVs make charging requests simultaneously; and  $z_{ij}^s$  and  $z_{ij}^d$  are the numbers of BEVs reaching and leaving charging station  $j$ , respectively, during period  $t_{ij}^{os}$ .  $z_j^o$  can be immediately obtained through information services about charging stations. Moreover, we assume that the average BEV arrival rate for charging station  $j$  is  $\lambda_j$ , and  $z_{ij}^s$  is equal to  $\lambda_j t_{ij}^{os}$  during period  $t_{ij}^{os}$ . Let  $y_{jk}^0$  ( $k=1,2,\dots,h_j$ ) denote the energy of the BEVs being charged in charger  $k$  when the target BEVs make charging requests simultaneously. In this case,  $h_j$  is the number of chargers in charging station  $j$ . In general, BEV drivers will begin charging even when their BEVs are not yet about to run out of energy to protect battery and alleviate range anxiety (Sun et al., 2015). By contrast, when the energy of batteries is less than a fixed value, the power warning system of BEVs can remind drivers to charge their batteries. Therefore, the mean energy at charging initiation can be reasonably assumed as a fixed value, with  $e'$  denoting the fixed value. Moreover, drivers do not necessarily fully charge their BEVs because of the long charging time and the aim to reach their destinations as quickly as possible. When a BEV is charged, its battery energy can reach a certain value within a relatively short period, which is generally sufficient to reach its destination (Yong et al., 2015). Therefore, we assume that the mean energy at charging termination is the aforementioned value, and let  $\bar{e}$  denote this value. The average charging amount of BEVs is  $\Delta e = \bar{e} - e'$ . Let  $p_j$  denote the charging power of the chargers in charging station  $j$ . During period  $t_{ij}^{os}$ , the total charging amount of charger  $k$  is equal to  $p_j t_{ij}^{os}$  kWh. Let  $z_{ijk}^d$  denote the number of BEVs leaving charger  $k$  in charging station  $j$  during period  $t_{ij}^{os}$ . For charger  $k$ , if  $p_j t_{ij}^{os} < \bar{e} - y_{jk}^0$ , then no BEV leaves charger  $k$  and  $z_{ijk}^d = 0$ . If  $p_j t_{ij}^{os} \geq \bar{e} - y_{jk}^0$ , then at least one BEV departs from charger  $k$  during period  $t_{ij}^{os}$ , and the number of BEVs leaving charger  $k$  is expressed as

$$z_{ijk}^d = INT\left(\frac{p_j t_{ij}^{os} + y_{jk}^0 - \bar{e}}{\Delta e}\right) + 1, \quad (6)$$

where  $INT(b)$  is the maximum integer less than or equal to  $b$ .

The number of BEVs departing from charging station  $j$  is

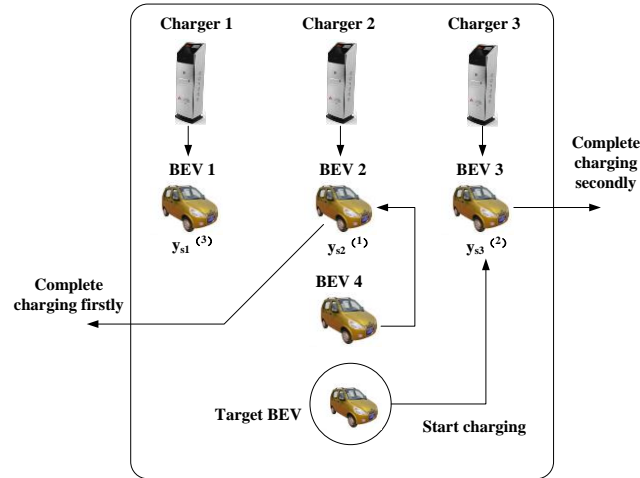
$$z_{ij}^d = \sum_{k=1}^{h_j} z_{ijk}^d. \quad (7)$$

Therefore,  $z_{ij}$  is obtained because  $z_j^o$ ,  $z_{ij}^s$  and  $z_{ij}^d$  are determined. If  $z_{ij} < h_j$ , then  $t_{ij}^q = 0$ , as shown in

Eq. (2). However, if  $z_{ij} \geq h_j$ , then  $t_{ij}^q > 0$ . Let  $y_{ijk}$  be the energy of the BEV charged in charger  $k$  at charging station  $j$  when target BEV  $i$  reaches this charging station. To determine  $t_{ij}^q$  when  $z_{ij} \geq h_j$ ,  $y_{ijk}$  must be estimated. The estimated value of  $y_{ijk}$  is

$$y_{ijk} = P_j^{os} \frac{0}{t_j^k} y_j^k \frac{d}{z_{ij}} \epsilon \Delta. \quad (8)$$

For all the chargers  $k$  ( $k=1,2,\dots,h_j$ ) in charging station  $j$ ,  $y_{ijk}$  are arranged in descending order to determine the charger used to charge target BEV  $i$ . Let  $y_{ijk}^{(r)}$  ( $r=1,2,\dots,h_j$ ) denote the energy of the BEVs charged in charger  $k$  with serial number  $r$  in descending order. For example, if  $r_1 > r_2$ , then  $y_{ijk_1}^{(r_1)} < y_{ijk_2}^{(r_2)}$ . In particular, the BEV charged in charger  $k_2$  with  $y_{ijk_2}^{(r_2)}$  completes charging firstly, and then the BEV in charger  $k_1$  completes charging subsequently. The estimated serial number of charger  $k$  used to charge target BEV  $i$  is  $r = \text{MOD}(\frac{z_{ij}}{h_j}) + 1$ , where  $\text{MOD}(\frac{z_{ij}}{h_j})$  is the remainder of  $\frac{b_1}{b_2}$ . For example, if  $z_{ij}=4$  and  $h_j=3$ , then  $r = \text{MOD}(\frac{4}{3}) + 1 = 2$ , and target BEV  $i$  uses charger  $k$  with serial number  $r = 2$  to charge its battery. Fig. 2 presents the queuing process of the target BEV in a charging station.



**Fig. 2.** Queuing process in the charging station.

Eq. (3) presents the charging time of target BEV  $i$  in charging station  $j$ . The main factors that affect charging time are charging amount and  $p_j$ .  $p_j$  can be obtained through the information of the charging station. However, the charging amount of target BEV  $i$  charging at charging station  $j$  must be estimated. When target BEV  $i$  reaches charging station  $j$ , the remaining energy  $e_{ij}^s$  is

$$e_{ij}^s = E_i^0 - \gamma_{ij}^{os}, \quad (9)$$

where  $E_i^0$  is the initial energy of target BEV  $i$  and  $\gamma_{ij}^{os}$  is the energy consumption of target BEV  $i$  driving from its departure point to charging station  $j$ , which is determined based on Eq. (13) presented in the following subsection.

To ensure that target BEV  $i$  will successfully reach its destination without depleting its energy en route, its battery energy at charging completion in charging station  $j$  must be greater than the energy consumption of BEV  $i$  from charging station  $j$  to its destination. Moreover, when target BEV  $i$  reaches its destination, its battery energy cannot be depleted because of driver range anxiety. Let  $\varepsilon^d$  denote the lower limit of battery energy at the destination. To obtain minimum charging time, we assume that when target BEV  $i$  reaches its destination, the remaining energy is equal to  $\varepsilon^d$ . Therefore, when target BEV  $i$  completes charging at charging station  $j$ , its energy  $e_{ij}^{\bar{s}}$  is given by

$$e_{ij}^{\bar{s}} = \gamma_{ij}^{sd} + \varepsilon^d, \quad (10)$$

where  $\gamma_{ij}^{sd}$  is the energy consumption of target BEV  $i$  driving from charging station  $j$  to its destination, which is determined based on Eq. (14) as shown in the following subsection.

Eqs. (4) and (5) present the driving times for the two sub-routes. Let  $A$  denote the set of all links  $a$  in the network. In general, the driving time of a BEV traversing link  $a \in A$  is a strictly increasing function of traffic flow on link  $a$ . The Bureau of Public Roads (BPR) function can be adopted to present the relation between driving time and traffic flow (He et al., 2013). For example, the following form of BPR function can be used:

$$t_a = t_a^0 \left[ 1 + 0.15 \left( \frac{v_a}{c_a} \right)^4 \right], \quad (11)$$

where  $t_a$  is the driving time of a BEV traversing link  $a$ ,  $t_a^0$  is the free-flow driving time of link  $a$ ,  $c_a$  is the capacity of link  $a$  and  $v_a$  is the traffic flow of link  $a$ .

### 2.2.2. Minimising energy consumption

The energy consumption between the departure point and the destination is also an important factor that influences route and charging station choices. The objective function for minimising energy consumption can be given as follows:

$$\min U = \min \sum_{i=1}^m (\gamma_{ij}^{os} + \gamma_{ij}^{sd}) x_{ij}, \quad \dots \quad (12)$$

where

$$\gamma_{ij}^{os} = \sum_{a \in A_{ij}^{os}} \gamma_a, \quad (13)$$

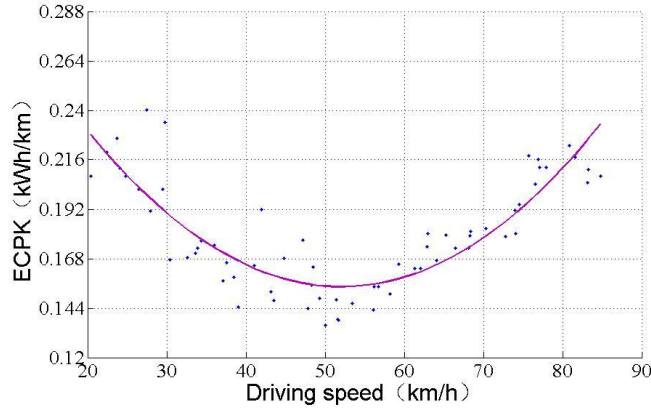


$$\gamma_{ij}^{sd} = \sum_{a \in A_{ij}^{sd}} \gamma_a \cdot \dots \cdot \dots \cdot \dots \quad (14)$$

Objective (12) aims to minimise the total energy consumption  $U$  of all target BEVs, which is a zero–one integer programming, where  $\gamma_{ij}^{os}$  and  $\gamma_{ij}^{sd}$  are the energy consumption levels of target BEV  $i$  driving from its departure point to charging station  $j$  and from charging station  $j$  to its destination, respectively.

Eqs. (13) and (14) present the energy consumption of target BEV  $i$  traversing the two aforementioned sub-routes, respectively, where  $\gamma_a$  is the energy consumption when a BEV traverses link  $a$ , and  $A_{ij}^{os} \in A$  and  $A_{ij}^{sd} \in A$  are the set of links traversed by target BEV  $i$  from its departure point to charging station  $j$  and from charging station  $j$  to its destination.

In general, energy consumption is typically proportional to the driving distance with fixed ECPK, and ECPK depends on driving speed. For both BEV and ICEV, energy consumption increases as driving distance increases with fixed driving speed. However, with fixed driving distance, the change trend of BEV energy consumption differs from that of ICEV energy consumption when driving speed changes. The relation between ECPK and BEV driving speed is nonlinear (Gardner et al., 2013). To obtain  $\gamma_a$ , data from BEVs operating in Beijing are adopted to explore the relation between ECPK and driving speed. The data are collected from 70 BEVs that are widely used in Beijing and other similar cities. The BEVs operate in the road network as regular vehicles and the data are obtained online through an internal controller area network bus. Thus, the experimental data have significant representativeness, which can ensure the practicability of experimental results. The experimental data include 65 complete discharging processes of BEV battery and driving speed. Fig. 3 presents the fitting result of the relation between ECPK and driving speed.



**Fig. 3.** Relation between driving speed and ECPK based on experimental data.

The average driving speed  $s_a$  can be obtained as follows:

$$s_a = \frac{l_a}{t_a}, \quad (15)$$

where  $l_a$  is the length of link  $a$ . From the fitting result shown in Fig. 3, ECPK is expressed as

$$\Delta\gamma_a = 7.344 \times 10^{-5} \times s_a^2 - 7.656 \times 10^{-3} \times s_a + 0.3536, \quad \dots \quad (16)$$

where  $\Delta\gamma_a$  refers to ECPK when a BEV operates on link  $a$ . Moreover, the determinate coefficient of the fitting result is 0.735, which indicates that the result demonstrates a good fitting effect (Yao et al., 2014). It is noted that, based on the experimental data as shown in Fig. 3, the unit of  $s_a$  in Eq. (16) is km/h and  $\Delta\gamma_a$  is kWh/km. Different from  $\Delta\gamma_a$ ,  $\gamma_a$  is the energy consumption when a BEV traverses link  $a$ , which is determined by the length of link  $a$   $l_a$ , besides  $\Delta\gamma_a$ .  $\gamma_a$  can be obtained as follows:

$$\gamma_a = \Delta\gamma_a l_a. \quad (17)$$

In Eq. (17), considering  $\Delta\gamma_a$  being obtained by Eq. (16) and its unit being kWh/km, the unit of  $l_a$  is km. Thus, the unit of  $\gamma_a$  in Eq. (17) is kWh.

### 2.2.3. Minimising charging costs

Another important factor that influences route and charging station choice is the cost of charging batteries in charging stations, which includes electricity cost, service cost and parking fee. The objective function for minimising charging cost is as follows:

$$\min C = \min \sum_{i=1}^m (c_{ij}^e + c_{ij}^s + c_{ij}^p) x_{ij}, \quad \dots \quad (18)$$

where

$$c_{ij}^e = \eta_j^e (e_{ij}^{\bar{s}} - e_{ij}^s), \quad \dots \quad (19)$$

$$c_{ij}^s = \eta_j^s (e_{ij}^{\bar{s}} - e_{ij}^s), \quad \dots \quad (20)$$

$$c_{ij}^p = \eta_j^p (t_{ij}^q - t_{ij}^c). \quad \dots \quad (21)$$

Objective (18) minimises the total charging cost  $C$  of all target BEVs, which is a zero–one integer programming, where  $c_{ij}^e$ ,  $c_{ij}^s$  and  $c_{ij}^p$  are the electricity cost, service cost and parking fee, respectively, of target BEV  $i$  charging in charging station  $j$ .

Eqs. (19) and (20) present the electricity and service costs of target BEV  $i$  charging in charging station  $j$ , which are proportional to the charging amount, where  $\eta_j^e$  and  $\eta_j^s$  are the unit electricity and unit service costs, respectively. These parameters can be immediately obtained through charging station information.

Eq. (21) calculates for the parking fee of target BEV  $i$  charging in charging station  $j$ , which is

proportional to the sum of queuing and charging times, where  $\eta_j^p$  is the unit parking fee of charging station  $j$ , which can be immediately obtained through charging station information.

### 2.3. Constraints

Several constraints on the route choice of target BEVs are illustrated to ensure operating safety and complete travel.

For target BEV  $i$ , if charging station  $j$  is chosen to charge its battery ( $x_{ij} = 1$ ), then  $e_{ij}^s$  should not be less than zero to ensure that BEV  $i$  can successfully reach charging station  $j$ . Moreover, if  $x_{ij} = 0$ , then  $e_{ij}^s$  is not considered, as shown in Eq. (22):

$$e_{ij}^s x_{ij} \geq 0 \quad (i=1, \dots, m; j=1, \dots, n). \quad (22)$$

Let  $E$  denote the nominal capacity of BEV batteries. To ensure that target BEV  $i$  can successfully reach its destination after charging at charging station  $j$ ,  $e_{ij}^{\bar{s}}$  should not exceed  $E$ . Moreover, if  $x_{ij} = 0$ , then  $e_{ij}^{\bar{s}}$  is not considered, as shown in Eq. (23):

$$e_{ij}^{\bar{s}} x_{ij} \leq E \quad (i=1, \dots, m; j=1, \dots, n). \quad (23)$$

To satisfy the travel time demand of the driver, the travel time of target BEV  $i$  must not be greater than the upper limit of travel time  $\tau_i$ , as shown in Eq. (24):

$$(t_{ij}^q + t_{ij}^c + t_{ij}^{os} + t_{ij}^d) x_{ij} \leq \tau_i \quad (i=1, \dots, m; j=1, \dots, n). \quad (24)$$

Let  $V_j$  denote the maximum number of BEVs that can stop over at charging station  $j$ . To ensure that target BEV  $i$  can choose charging station  $j$  to charge its battery, the number of BEVs in charging station  $j$ , including target and other BEVs, must not be greater than  $V_j$ , as shown in Eq. (25):

$$z_{ij} + \sum_{i=1}^m x_{ij} \leq V_j \quad (j=1, \dots, n). \quad (25)$$

For each target BEV, only one charging station can be chosen to charge its battery, as shown in Eq. (26):

$$\sum_{j=1}^n x_{ij} = 1 \quad (i=1, \dots, m). \quad (26)$$

Eq. (27) ensures that the decision variable  $x_{ij}$  is the binary variable.

$$x_{ij} \in \{0,1\} \quad (i=1,\dots,m; j=1,\dots,n) \quad (27)$$

Notably, in previous literature, the constraints mainly consider the limited driving range, battery capacity and charging station locations. Besides these conventional constraints, other constraints are considered in the model. The difference between the constraints in the proposed model and the previous literature lies in the following two aspects. Firstly, the travel time demand of the individual target BEV is considered in the constraints when the total travel times of all target BEVs reach the optimal value, as shown in Eq. (24). Secondly, the constraints further consider the interactions of different BEVs' charging behaviour, which is critical for the route choice problems when considering multiple target BEVs, as shown in Eq. (25).

### 3. Model transformation and solution

#### 3.1. Model transformation

The proposed model is a multi-objective optimisation problem (MOP). In general, the optimal solution for MOP cannot make all the objective functions obtain the optimal solutions simultaneously. Conventional solution methods cannot be adopted to solve this problem (Liu et al., 2014). To solve MOP, an effective method is to transform this problem into a model with a single objective function. However, the objective functions with different magnitudes cannot be directly combined by a deterministic approach, because the dimensions of the objective functions are not uniform. Therefore, FPA is applied to address the problem given the different magnitudes of each objective. This method adopts a membership function to quantify the fuzzy goals of the objective functions, and the solution is obtained objectively (Stanley, 2001). Through the membership function of FPA, the closeness of the obtained solution to the optimal solution for each objective function is determined, which has fuzzy properties, and the dimensions of the objective function are not considered in the outputs of the membership function. For the proposed multi-objective optimisation model, the objective functions include  $\min T$ ,  $\min U$  and  $\min C$ , as shown in Eqs. (1), (12) and (18), respectively. When  $\min T$  is selected as the example, the linear membership function (Wang et al., 2009) is used to describe the fuzzy goals of  $\min T$  to facilitate solution computation as follows:

$$u(T) = \begin{cases} 0 & T \geq T_{\max} \\ \frac{(T_{\max} - T)}{(T_{\max} - T_{\min})} & T_{\min} < T < T_{\max} \\ 1 & T \leq T_{\min} \end{cases} \quad (28)$$

where  $u(T)$  denotes the linear membership function of objective  $\min T$ , and  $T_{\min}$  and  $T_{\max}$  are the minimum and maximum values, respectively, of  $T$ .

Moreover, the trade-offs among different travelling cost components for BEV drivers are

important factors in choosing routes (Sun et al., 2016). Such trade-offs are described by the weighting coefficients of different objectives, which are used to transform several linear membership functions into an objective function. The importance degree relations among the objectives are considered, and fuzzy preference relations (FPR) are used to obtain the weighting coefficients of each objective (Chen et al., 2012). To present the trade-offs among different objectives for BEV drivers, let  $\ll$ ,  $<$ , and  $\approx$  denote the relations of extreme unimportance, unimportance and equal importance, respectively, between two different objectives. For example, the relative importance between objectives  $T$  and  $U$  is indicated as follows:

$$\begin{cases} \text{if } T \ll U & \text{then } \varphi_{TU} = \alpha, \varphi_{UT} = 1 - \alpha \\ \text{if } T < U & \text{then } \varphi_{TU} = \beta, \varphi_{UT} = 1 - \beta \quad (0 < \alpha < \beta < 0.5), \\ \text{if } T \approx U & \text{then } \varphi_{TU} = \varphi_{UT} = 0.5 \end{cases} \quad (29)$$

where  $\varphi_{TU}$  and  $\varphi_{UT}$  are the corresponding relative importance coefficients, which range from 0 to 1, and  $\varphi_{TU} + \varphi_{UT} = 1$ ;  $\alpha$  and  $\beta$  are the parameters that represent the values of  $\varphi_{TU}$  and  $\varphi_{UT}$ , respectively, when the relation between two different objectives is not equally important. When the relation is equally important, the values of  $\varphi_{TU}$  and  $\varphi_{UT}$  are equal to 0.5. For objective  $T$ , the summation of the relative importance coefficients is

$$S_R(T) = \varphi_{TU} + \varphi_{TC}, \quad (30)$$

where  $S_R(T)$  is the summation of the relative importance coefficients between  $T$  and other objectives.

The weighting coefficient of objective  $T$  is as follows:

$$W(T) = \frac{S_R(T)}{S_R(T) + S_R(U) + S_R(C)}, \quad (31)$$

where  $W(T)$  is the weighting coefficient of objective  $T$ .

Furthermore, given the linear membership functions  $u(T)$ ,  $u(U)$  and  $u(C)$ , and the weighting coefficients of the objectives  $W(T)$ ,  $W(U)$  and  $W(C)$ , the proposed model can be transformed into a model with a single objective function to obtain the maximum value as follows:

$$Z = \max [(W(T)\mu(T))^p + (W(U)\mu(U))^p + (W(C)\mu(C))^p]^{\frac{1}{p}}, \quad (32)$$

where  $Z$  is the objective function value, and  $p$  is the distance coefficient. The possible values of  $p$  can be determined based on the following situations.

*Situation 1:*  $p=1$ . The objective function is to obtain the maximum value of the linear weighted sum of the three objectives, namely Manhattan distance, as follows:

$$Z_{p=1} = \max [W(T)\mu(T) + W(U)\mu(U) + W(C)\mu(C)]. \quad (33)$$

*Situation 2:*  $p=2$ . The objective function is to obtain the maximum value of the square root of the

quadratic weighted sum of the three objectives, namely Euclidean distance, as follows:

$$Z_{p=2} = \max [(W(T)\mu(T))^2 + (W(U)\mu(U))^2 + (W(C)\mu(C))^2]^{\frac{1}{2}}. \quad (34)$$

*Situation 3:  $p = \infty$ .* The objective function is to obtain the maximum value of the objective with minimum weighted value among the three objectives, namely Chebyshev distance, as follows:

$$Z_{p=\infty} = \max [\min\{W(T)\mu(T), W(U)\mu(U), W(C)\mu(C)\}]. \quad (35)$$

### 3.2. Model solution

Through model transformation, a multi-objective optimisation model with a single objective function is obtained. This problem is a complex nonlinear optimisation problem. Adopting an effective algorithm to solve the problem is highly important. A GA is a stochastic optimisation procedure that can solve optimisation problems in different situations. Therefore, a GA is designed to obtain the optimal solution, i.e. to determine the routes with minimum comprehensive travelling cost for target BEVs.

A GA records the parameters of the problem into the chromosome. For the proposed model, the serial numbers of the charging stations, which are chosen by the drivers of target BEVs to charge their vehicles, are chosen as genes for any chromosome. The number of chromosomes is defined as the population size. A chromosome represents the choice result of the charging stations for the target BEVs. A chromosome is formed by the  $1 \times m$  vector, where  $m$  is the number of target BEVs in a road network, and the value of each gene in the chromosome is the serial number of the charging station that is chosen by the driver of corresponding target BEV. The fitness functions are the objective functions shown in Eqs. (33)–(35). An iterative operation is applied to optimise the chromosomes through selection, intercross and mutation operations. The detailed operation steps are described in the relevant literature (Shafahi, et al., 2010). Moreover, to ensure that the solutions satisfy the constraints of the model, a check operation is applied to determine the values of objective functions under each chromosome in each iterative operation after intercross and mutation operations. The check operation considers the constraints shown in Eqs. (22)–(25). If a chromosome does not satisfy at least one of the constraints, the values of objective functions under the chromosome are equal to zero. Otherwise, the values of objective functions are equal to their true values, which are greater than zero. Through the check operation, the chromosomes that do not satisfy the constraints are deleted from the solutions since the objective functions are to obtain the maximum value. The constraints shown in Eqs. (26)–(27) are considered and satisfied when forming the chromosomes. For GA termination, many studies have used three termination criteria to end the algorithm (Kang et al., 2015) as follows. (1) The optimal solution does not change after a given number of iterations. (2) The difference between the optimal and worst solutions in a population is less than a given value. (3) The iterations reach the maximum number. In this study, the third method is adopted as the termination criterion.

A GA is adopted to obtain the optimal solutions for the objective functions as shown in Eqs. (33)–(35), and the solution with the minimum comprehensive travelling cost among the three situations is selected as the final optimal solution for the proposed model. The comprehensive travelling cost cannot

be obtained immediately because of the varying dimensions of different travelling costs. However, the comprehensive travelling costs of different situations can be compared in pairs to determine the solution with the minimum comprehensive travelling cost. The comparison method aims to obtain the weighted summation of the ratios of the solution values  $T$ ,  $U$  and  $C$  between two situations based on the weighting coefficients. For example, to compare the comprehensive travelling costs of situations  $p = 1$  and  $p = 2$ , the results are

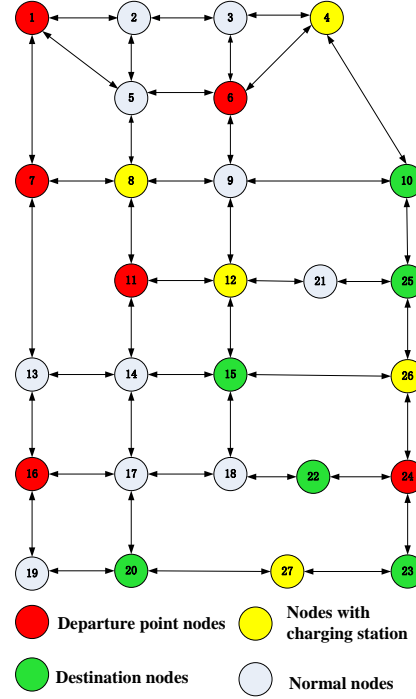
$$\xi\left(\frac{Z_{p=1}}{Z_{p=2}}\right) = W \left( \frac{T_{(Z_{p=1})}}{T_{(Z_{p=2})}} \right) + W \left( \frac{U_{(Z_{p=1})}}{U_{(Z_{p=2})}} \right) + W \left( \frac{C_{(Z_{p=1})}}{C_{(Z_{p=2})}} \right), \quad (36)$$

where  $\xi\left(\frac{Z_{p=1}}{Z_{p=2}}\right)$  is the comparison coefficient between situations  $p = 1$  and  $p = 2$ , and its value is compared with 1. If the value is less than 1, then the comprehensive travelling cost of situation  $p = 1$  is less than that of situation  $p = 2$ ; otherwise, the relation is not achieved.  $T_{(Z_{p=1})}$ ,  $U_{(Z_{p=1})}$  and  $C_{(Z_{p=1})}$  present the  $T$ ,  $U$  and  $C$  values, respectively, under the objective function  $Z_{p=1}$ , and  $T_{(Z_{p=2})}$ ,  $U_{(Z_{p=2})}$  and  $C_{(Z_{p=2})}$  are the  $T$ ,  $U$  and  $C$  values, respectively, under the objective function  $Z_{p=2}$ .

## 4. Numerical example

### 4.1. Example scenario description

In this section, a numerical example is presented to demonstrate the proposed model. The model is applied to solve a route choice problem for multiple BEVs in the road network that consists of 27 nodes and 84 links (Fig. 4). In this numerical example, 6 target BEVs and 5 charging stations exist in the road network. Among the nodes of the road network, 5 nodes (yellow) have charging stations. In addition, 6 departure point nodes (red) and 6 destination nodes (green) are present. The other 10 nodes (grey) are the normal nodes that may be passed by the target BEVs.



**Fig. 4.** Road network for the numerical example.

The departure point nodes are numbered 1, 6, 7, 11, 16 and 24. The destination nodes are numbered 10, 15, 20, 22, 23 and 25. The O-D pair are  $1 \rightarrow 10$ ,  $6 \rightarrow 25$ ,  $7 \rightarrow 15$ ,  $11 \rightarrow 22$ ,  $16 \rightarrow 23$  and  $24 \rightarrow 20$ . The initial energy of the target BEVs at nodes 1, 6, 7, 11, 16 and 24 are 11.11, 11.55, 7.81, 11.58, 10.24 and 7.67 kWh, respectively. The nominal capacity of the batteries is  $E = 24$  kWh. The nodes with charging station are numbered 4, 8, 12, 26 and 27. Table 1 lists the given parameters of the charging stations at each node, which includes charger number  $h_j$ , unit electricity cost  $\eta_j^e$  (\$/kWh), unit service cost  $\eta_j^s$  (\$/kWh), unit parking fee  $\eta_j^p$  (\$/h), BEV number  $z_j^o$ , arrival rate  $\lambda_j$  (veh/min), charging power  $p_j$  (kWh/h) and maximum number of BEVs stopping over at charging station  $V_j$ . Moreover, when determining the values of the parameters, the actual experience and operating information regarding the charging stations in Beijing are considered. The logic behind the determination of the parameter values is as follows: The values of  $h_j$ ,  $z_j^o$  and  $p_j$  are randomly determined within the reasonable ranges based on the actual experience. The values of  $\eta_j^e$ ,  $\eta_j^s$  and  $\eta_j^p$  refer to the relevant price of charging stations in Beijing. The values of  $\lambda_j$  are determined based on the values of  $h_j$  and the assumption that the charging station with more chargers would attract more BEVs (not the target BEVs) to choose it. The values of  $V_j$  are randomly determined within the reasonable ranges and larger than  $h_j$  in each charging station.

**Table 1**

Parameters of the charging stations at each node.

Nodes	$h_j$	$\eta_j^e$	$\eta_j^s$	$\eta_j^p$	$z_j^o$	$\lambda_j$	$p_j$	$V_j$
4	3	0.2555	0.1221	0.3052	2	0.0423	50	16



8	10	0.2555	0.1221	0.3052	9	0.1429	40	18
12	4	0.2555	0.1221	0.3052	5	0.0571	50	16
26	10	0.2555	0.1221	0.3052	11	0.1429	30	20
27	4	0.2555	0.1221	0.9156	5	0.0571	40	18

For the BEVs being charged in the charging stations, with the exception of the target BEVs, we assume that the mean energy at charging initiation  $e'$  is equal to 7.2 kWh, and the mean energy at charging termination  $\bar{e}$  is equal to 19.2 kWh. Moreover, Table 2 presents the given parameters for capacity and free-flow travel time of each link.

**Table 2**

Capacity  $c_a$  ( $10^3$  veh/h) and free-flow travel time  $t_a^0$  (min) of each link.

Link	$c_a$	$t_a^0$	Link	$c_a$	$t_a^0$	Link	$c_a$	$t_a^0$
1-2	41.85	12.18	4-3	34.84	15.87	6-3	31.75	16.70
1-7	45.85	9.78	4-6	13.53	13.42	6-4	15.85	14.30
2-1	11.59	18.68	4-10	37.07	14.01	7-1	39.06	15.02
2-3	46.19	18.65	5-1	7.40	11.65	7-8	17.22	13.79
2-5	33.82	13.96	5-2	18.18	16.37	8-7	28.26	10.96
3-2	10.29	13.55	5-6	8.03	11.20	8-5	36.76	11.80
3-4	18.25	12.81	6-5	10.27	14.48	8-9	45.20	12.87
9-8	30.06	17.27	11-12	42.23	11.76	14-11	48.21	10.85
9-6	48.13	12.24	12-11	36.57	11.92	14-13	30.08	16.82
9-10	48.46	10.72	12-9	19.95	14.11	14-15	12.10	11.59
10-4	12.93	17.46	12-21	47.81	15.72	14-17	12.57	11.79
10-9	48.71	13.24	13-7	7.52	10.64	15-12	17.33	10.20
10-25	48.11	11.79	13-14	25.30	18.49	15-14	43.00	12.22
11-8	27.36	13.11	13-16	22.79	16.82	15-18	17.19	13.27
15-26	41.21	10.11	17-20	39.68	13.35	20-19	41.83	12.75
16-13	12.24	11.16	18-15	40.99	14.04	20-27	16.72	16.54
16-17	24.56	18.61	18-17	14.22	12.92	21-12	46.89	12.78
16-19	46.29	17.26	18-22	27.55	12.84	21-25	31.75	10.94
17-14	40.86	15.68	19-16	25.60	13.60	22-18	15.85	17.66
17-16	48.22	9.18	19-20	34.44	13.96	1-5	41.85	12.18
17-18	34.85	11.00	20-17	37.21	16.09	3-6	45.85	9.78
23-27	7.57	12.51	26-15	39.21	16.04	6-9	11.59	18.68
24-22	43.36	16.61	26-24	18.15	14.29	7-13	46.19	18.65
24-23	47.10	9.77	26-25	35.91	12.00	9-12	33.82	13.96
24-26	35.86	9.69	27-20	34.82	17.16	8-11	10.29	13.55
25-10	39.34	10.07	27-23	13.15	13.73	11-14	18.25	12.82
25-21	38.70	15.11	22-24	11.24	13.08	12-15	30.06	17.27
25-26	23.26	15.93	23-24	27.93	19.29	5-8	36.76	11.80

To determine the driving time of each link, the traffic volume on each link is required. Table 3 lists the given parameters of the traffic volume and length of each link.

**Table 3**Traffic volume  $v_a$  ( $10^3$  veh/h) and length  $l_a$  (km) of each link.

Link	$v_a$	$l_a$	Link	$v_a$	$l_a$	Link	$v_a$	$l_a$
1-2	14.6	14.17	4-3	18.4	16.48	6-3	43.92	18.76
1-7	9.01	10.50	4-6	2.24	14.51	6-4	3.81	15.50
2-1	2.91	19.03	4-10	22.31	15.47	7-1	8.38	16.22
2-3	28.45	19.45	5-1	1.95	12.96	7-8	2.31	15.87
2-5	16.01	14.91	5-2	11.89	17.45	8-7	5.31	12.08
3-2	3.62	14.89	5-6	5.54	11.89	8-5	14.97	13.01
3-4	15.16	13.38	6-5	7.69	16.87	8-9	16.38	14.71
9-8	17.59	19.00	11-12	19.03	11.84	14-11	20.21	12.30
9-6	26.46	13.69	12-11	3.07	13.68	14-13	6.55	18.44
9-10	44.44	11.11	12-9	4.57	16.26	14-15	41.12	11.95
10-4	3.70	17.80	12-21	43.67	17.80	14-17	18.71	12.26
10-9	36.88	13.90	13-7	1.15	10.81	15-12	4.25	11.71
10-25	36.27	12.42	13-14	20.90	19.29	15-14	6.45	12.28
11-8	10.41	14.04	13-16	12.27	17.76	15-18	6.96	14.36
15-26	23.40	10.96	17-20	39.53	14.87	20-19	3.27	13.11
16-13	0.93	11.32	18-15	3.20	14.36	20-27	4.12	19.23
16-17	1.32	19.42	18-17	6.30	14.47	21-12	5.16	14.30
16-19	24.57	19.56	18-22	2.94	13.06	21-25	3.07	11.85
17-14	31.83	15.75	19-16	24.63	15.09	22-18	11.25	19.05
17-16	45.03	10.60	19-20	0.16	15.11	1-5	14.65	14.17
17-18	4.53	12.35	20-17	28.84	18.18	3-6	9.01	10.50
23-27	4.31	13.53	26-15	32.04	17.95	6-9	2.91	19.03
24-22	20.35	18.21	26-24	15.76	16.44	7-13	28.45	19.45
24-23	0.56	10.15	26-25	3.03	13.79	9-12	16.01	14.91
24-26	12.09	10.43	27-20	13.92	18.12	8-11	3.62	14.89
25-10	6.38	11.69	27-23	3.42	15.33	11-14	15.17	13.38
25-21	30.74	16.49	22-24	8.99	13.51	12-15	17.59	19.00
25-26	7.24	17.32	23-24	12.05	19.39	5-8	14.97	13.69

For the target BEVs, we assume that the remaining energy in the destination  $\epsilon^d$  is equal to 7.2 kWh. The upper limit of the total travel time  $\tau_i$  is 1200 min. Given Eq. (11), the driving time when a BEV traverses each link are obtained. Then, based on the data fitting result regarding the relation between driving speed and ECPK, as shown in Fig. 3, the energy consumption when a BEV traverses each link are obtained using Eqs. (15)–(17). Table 4 lists the driving time and energy consumption of each link  $a$  in the road network.

**Table 4**Driving time  $t_a$  (min) and energy consumption  $\gamma_a$  (kWh) of each link  $a$ .

Link	$t_a$	$\gamma_a$	Link	$t_a$	$\gamma_a$	Link	$t_a$	$\gamma_a$
------	-------	------------	------	-------	------------	------	-------	------------

1-2	12.21	2.50	4-3	16.05	2.64	6-3	18.41	3.00
1-7	9.78	1.73	4-6	13.41	2.40	6-4	14.31	2.58
2-1	18.69	3.04	4-10	14.29	2.57	7-1	15.03	2.69
2-3	19.05	3.11	5-1	11.65	2.20	7-8	13.79	2.78
2-5	14.06	2.44	5-2	16.82	2.82	8-7	10.96	2.03
3-2	13.58	2.50	5-6	11.58	1.90	8-5	12.81	2.08
3-4	13.73	2.10	6-5	15.16	2.86	8-9	13.08	2.52
9-8	17.56	3.15	11-12	11.83	1.88	14-11	11.00	2.10
9-6	12.40	2.31	12-11	11.93	2.39	14-13	16.83	3.09
9-10	11.85	1.73	12-9	14.11	2.85	14-15	12.52	1.86
10-4	17.48	2.85	12-21	17.36	2.86	14-17	12.05	1.96
10-9	13.90	2.20	13-7	10.64	1.73	15-12	10.22	2.04
10-25	12.36	1.97	13-14	19.78	3.03	15-14	12.35	1.94
11-8	13.15	2.31	13-16	17.03	2.88	15-18	13.33	2.38
15-26	10.27	1.80	17-20	15.33	2.33	20-19	12.75	2.11
16-13	11.16	1.81	18-15	14.04	2.30	20-27	16.54	3.40
16-17	18.60	3.15	18-17	12.99	2.46	21-12	12.78	2.44
16-19	17.46	3.34	18-22	12.84	2.09	21-25	10.94	1.97
17-14	16.54	2.46	19-16	15.35	2.38	22-18	17.67	3.16
17-16	10.22	1.71	19-20	13.96	2.50	1-5	12.21	2.50
17-18	11.01	2.11	20-17	16.96	3.00	3-6	9.78	1.73
23-27	12.71	2.22	26-15	17.11	2.92	6-9	18.69	3.04
24-22	16.73	3.03	26-24	15.51	2.69	7-13	19.05	3.11
24-23	9.77	1.64	26-25	12.00	2.40	9-12	14.06	2.44
24-26	9.71	1.72	27-20	17.23	2.95	8-11	13.58	2.50
25-10	10.07	2.06	27-23	13.74	2.61	11-14	13.73	2.10
25-21	16.02	2.65	22-24	13.88	2.12	12-15	17.57	3.15
25-26	15.96	2.88	23-24	19.39	3.08	5-8	12.81	2.08

#### 4.2. Optimal results and analysis

To analyse the influences of various driver trade-offs among different objectives on the results, four weighting conditions are considered in the numerical example. Table 5 lists the relative importance among the objectives under each weighting condition.

**Table 5**

Relative importance of the objectives under four weighting conditions.

Weighting conditions	Relative importance of the objectives
Condition 1	$C \approx U, C \approx T, U \approx T$
Condition 2	$C < 'U, C < 'T, U < 'T$
Condition 3	$T < 'U, T < 'C, U < 'C$
Condition 4	$T < 'C, T < 'U, C < 'U$

Given the relative importance of the objectives, the weighting coefficient of each objective under

the four weighting conditions can be obtained using Eqs. (29)–(31). When the linear membership function of each objective is combined, as shown in Eq. (28), the MOP is transformed into a model with a single objective function, i.e. Eq. (32), which comprehensively considers three objectives. The GA is designed to solve the problem. The given parameters of the GA are population size (50) and generation (100). The generation gap value is 0.9, which is a parameter for the selection operation. The generation gap value represents the proportion of the selected chromosomes in the population size. It is used to determine the number of chromosomes that are selected for intercross and mutation operations. The probabilities of intercrossing and mutation are 0.9 and 0.05, respectively. The results under different  $p$  values are compared using Eq. (36), and the optimal results of each weighting condition are presented in Table 6.

**Table 6**  
Solutions of the numerical example under four weighting conditions.

Weighting conditions	Values of $p$	$T$ (min)	$U$ (kWh)	$C$ (\$)
Condition 1	$p = 2$	562.00	57.82	22.68
Condition 2	$p = 1$	489.85	53.41	24.27
Condition 3	$p = \infty$	677.37	54.71	21.13
Condition 4	$p = 2$	525.95	53.37	23.25

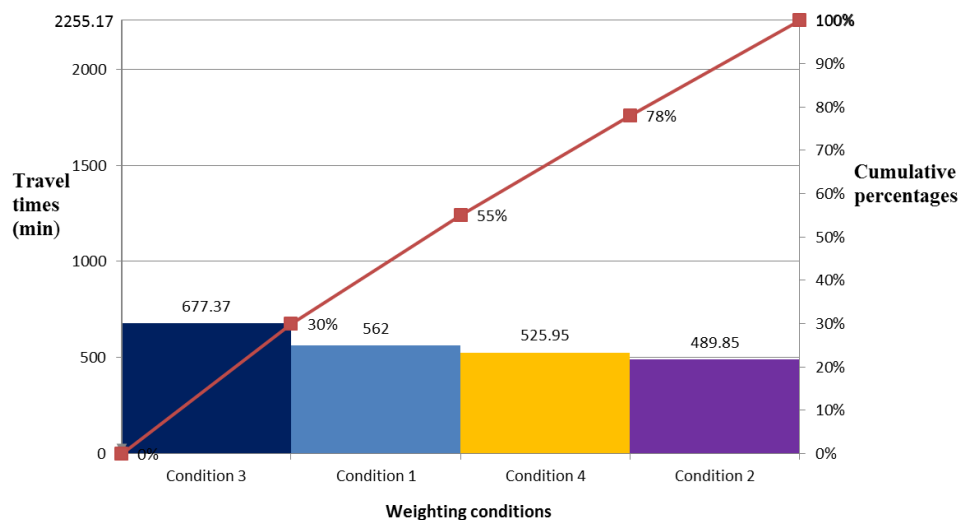
Table 7 lists the optimal routes under the solutions of each weighting condition. For each solution, the optimal route sequences from departure points to charging stations and from charging stations to destinations are determined based on the *Assumption 1* as mentioned in Section 2.1. When determining the  $\gamma_a$  of each link, the routes with minimum energy consumption can be obtained using a shortest path algorithm, such as Dijkstra algorithm (Dijkstra, 1959), and the values of  $\gamma_a$  are regarded as the weight of link  $a$ . After determining the optimal route sequences and corresponding energy consumption values, the model that is comprised of Eq. (32) and Eqs. (22)–(27) is solved with the GA and the optimal charging stations for the target BEVs are obtained. The serial number of the optimal charging stations is recorded in the chromosomes of the optimal solutions, which is marked as “charging” in Table 7.

**Table 7**  
Optimal routes of the numerical example under four weighting conditions.

Weighting conditions	Departure points of target BEVs	Routes
Condition 1	1	1 → 5 → 6 → 4(charging) → 10
	6	6 → 4(charging) → 10 → 25
	7	7 → 8(charging) → 11 → 14 → 15
	11	11 → 12(charging) → 15 → 18 → 22
	16	16 → 17 → 20 → 27(charging) → 23
	24	24 → 26(charging) → 24 → 23 → 27 → 20
Condition 2	1	1 → 5 → 6 → 4(charging) → 10
	6	6 → 4(charging) → 10 → 25
	7	7 → 8(charging) → 11 → 14 → 15

Condition 3	11	11 → 12(charging) → 15 → 18 → 22
	16	16 → 17 → 20 → 27(charging) → 23
	24	24 → 26(charging) → 24 → 23 → 27 → 20
	1	1 → 7 → 8(charging) → 9 → 10
	6	6 → 4(charging) → 10 → 25
	7	7 → 8(charging) → 11 → 14 → 15
	11	11 → 12(charging) → 15 → 18 → 22
Condition 4	16	16 → 13 → 14 → 15 → 26(charging) → 24 → 23
	24	24 → 23 → 27(charging) → 20
	1	1 → 7 → 8(charging) → 9 → 10
	6	6 → 4(charging) → 10 → 25
	7	7 → 8(charging) → 11 → 14 → 15
	11	11 → 12(charging) → 15 → 18 → 22
	16	16 → 17 → 20 → 27(charging) → 23
	24	24 → 23 → 27(charging) → 20

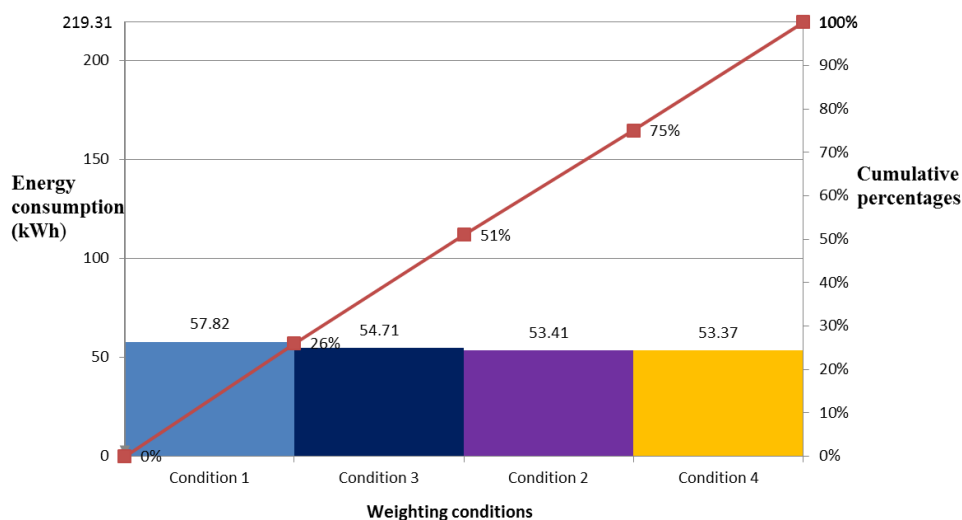
The Pareto curves are determined based on the solutions of the numerical example under the four weighting conditions to explore the influences of various driver trade-offs among different objectives on each objective. Fig. 5 presents the travel times for the numerical example under each weighting condition and the corresponding cumulative percentages. For travel times, the solution for Condition 2 holds the minimum value and that for Condition 3 holds the maximum value. The gap between the minimum and maximum values is 187.52 min. Moreover, the cumulative percentages indicate that the travel time under Condition 3 accounts for 30%; however, the travel times under Conditions 1, 2 and 4 exhibit moderate changes in degree.



**Fig. 5.** Pareto curve of travel times under four weighting conditions.

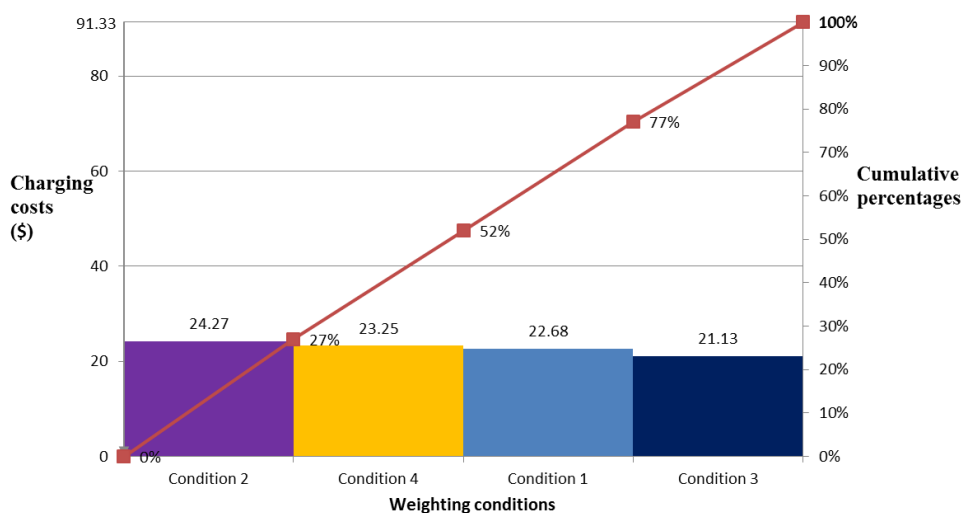
The energy consumption of the numerical example under each weighting condition and the corresponding cumulative percentages are shown in Fig. 6. The gap between the minimum and maximum values of energy consumption is 4.45 kW. The solution for Condition 4 holds the minimum value and that for Condition 1 holds the maximum value. Moreover, the cumulative

percentages indicate that the energy consumption under the four weighting conditions exhibits a moderate degree of change.



**Fig. 6.** Pareto curve of energy consumption under four weighting conditions.

Fig. 7 compares the charging costs of the numerical example under each weighting condition and the corresponding cumulative percentages. For charging costs, the solution for Condition 3 holds the minimum value and that for Condition 2 holds the maximum value. The gap between the minimum and maximum values is \$3.14. Moreover, the cumulative percentages indicate that charging costs under the four weighting conditions exhibit a moderate degree of change.



**Fig. 7.** Pareto curve of charging costs under four weighting conditions.

#### 4.3. Analysis of GA parameter and convergence test

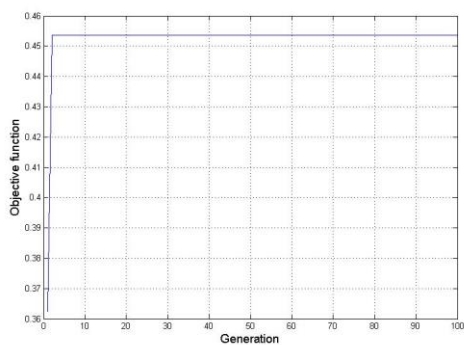
The results of the optimal route choices with multiple objectives for the numerical example show that the GA can be applied to solve the proposed model. For the GA, generation is an important parameter. Experiences show that a large generation tends to yield good results. Therefore, GA generation is tested in this section. In the numerical example, four weighting conditions are considered, and GA generation is set to 100. Through comparative analysis, the optimal results of Condition 1 are obtained under the objective function with  $p=2$ , as shown in Eq. (34). For Condition 2, the optimal results are obtained under the objective function with  $p=1$ , as shown in Eq. (33). For Condition 3, the optimal results are obtained under the objective function with  $p=\infty$ , as shown in Eq. (35). The optimal results of Condition 4 are obtained under the objective function with  $p=2$ . Let  $K$  denote GA generation. The values of the optimal objective functions under each weighting condition for each parameter  $K$  are presented in Table 8.

**Table 8**

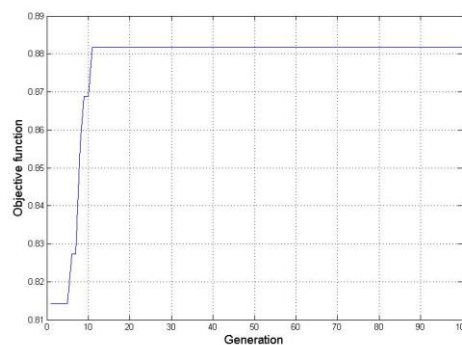
Different objective values under four weighting conditions that correspond to varying GA generations.

Weighting conditions	Values of $p$	$K = 0$	$K = 20$	$K = 40$	$K = 60$	$K = 80$	$K = 100$
Condition 1	$p = 2$	0	0.4129	0.4503	0.4536	0.4536	0.4536
Condition 2	$p = 1$	0	0.5147	0.7455	0.8817	0.8817	0.8817
Condition 3	$p = \infty$	0	0.2686	0.2855	0.3189	0.3189	0.3189
Condition 4	$p = 2$	0	0.4693	0.5176	0.5304	0.5304	0.5304

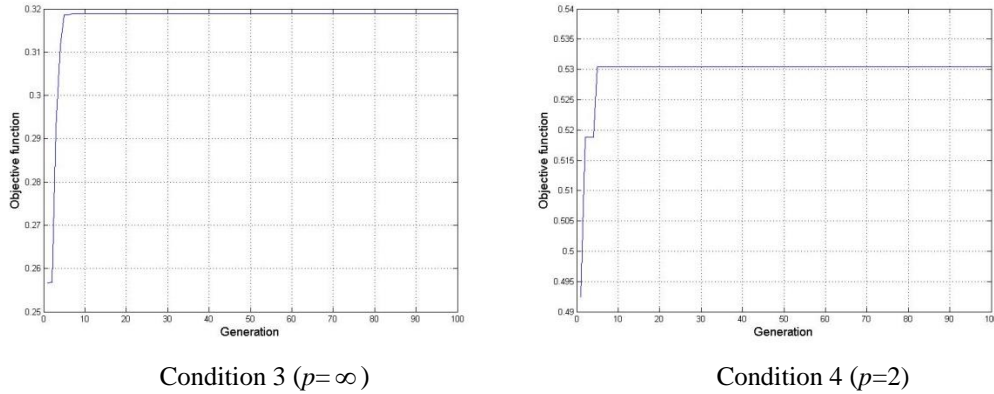
The test for GA generation demonstrates that generation  $K = 60$  results in a better solution. When  $K$  is larger than 60, the solution is constant. Moreover, a convergence test of the values of the optimal objective functions under each weighting condition is implemented to further test the efficiency of the GA solution in solving the proposed model in the numerical example. Fig. 8 presents the convergence processes of the values of the optimal objective functions under the four weighting conditions.



Condition 1 ( $p=2$ )



Condition 2 ( $p=1$ )



**Fig. 8.** Convergence test of the GA for the optimal objective functions under four weighting conditions.

We conclude that the GA demonstrates good efficiency for the problem based on the numerical example, and thus, is a feasible algorithm to solve the proposed model (Fig. 8).

## 5. Conclusions

The route choice problem in the travelling and charging of multiple BEVs is investigated in this study. A multi-objective optimisation model is proposed to explore the route choice problem in multiple BEVs by considering the limited driving range and charging behaviour of BEVs. The model has three objective functions, namely, minimising travel time, energy consumption and charging cost. Travel time includes queuing, charging and driving times. Moreover, the relation between ECPK and driving speed is formulated based on the data from BEVs operating in Beijing, which can be used to determine the energy consumption when a BEV traverses each link in a road network. Charging cost consists of electricity cost, service cost and parking fee. FPA and FPR are adopted to transform the three objective functions into a single objective function, which comprehensively considers the three optimisation objectives. The trade-offs among different travelling cost components for BEV drivers are considered in the transformed model. A GA is designed to obtain the optimal solution. Moreover, a numerical example is presented to demonstrate the model and solution algorithm, and four conditions of driver trade-offs among the different objectives are considered in the numerical example. The optimal choices under each condition for the target BEVs involved in the numerical example are determined. In addition, the influences of various driver trade-offs among different objectives on each objective value are explored via Pareto curves, and the effectiveness of the GA is verified through a convergence test.

Notably, the average charging amount of BEVs (excluding the target BEVs) is assumed based on BEV driving and charging experiences and is set as a fixed value in the model. However, the assumption overlooks the potential special conditions in charging BEVs. Therefore, built upon the proposed model, the dynamic charging process of BEVs will be further investigated in a future research based on substantial data, and the queuing time estimation of the target BEVs is projected to become increasingly accurate. Moreover, the assumption about the number of chargers in a charging station is not considered in this study. The assumption may make the model different, because it is related to the assumption about the capacity of a charging station, charging time and queuing time. In a future research, the assumption about the number of chargers in a charging station will be considered and its relations to the capacity of a charging station, charging time and queuing time will be explored to further investigate the problem of route choices for the travelling and charging of BEVs.



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