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Bi-objective programming approach for solving the metro timetable optimization problem with dwell time uncertainty

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ABSTRACT

For optimization of timetables in metro systems with regular cyclic operation, this paper develops a bi-objective programming approach addressed to minimization of net energy consumption and total travel time with provision for dwell time uncertainty. Firstly, we formulate the bi-objective timetable optimization problem as an expected value model with speed profile control. Secondly, we use the ε-constraint method within a genetic algorithm framework to determine the Pareto optimal solutions. Finally, numerical examples based on the real-life operation data from the Beijing Metro Yizhuang Line are presented in order to illustrate the practicability and effectiveness of the approach developed in the paper.

1. Introduction

1.1. Motivation

Due to their high reliability, large passenger capacity and environmental friendliness, metro systems have played an important role in many metropolitan cities (Jin et al., 2014; Yang et al., 2016a). Metro systems and metro timetables in particular have been a widely studied topic in recent years. The construction of a metro timetable includes scheduling the arrival and departure times of a set of trains at each station, and defining the operating speed profile of trains on each section. Speed profiles and travel times have substantial impacts of different kinds - on operational efficiency, on the convenience of passengers and on levels of pollutant emissions. For this reason, it is appropriate to treat timetable optimization as a multi-objective decision problem. Energy consumption and travel time are two important indices for evaluating the effectiveness of a timetable. The former is required by the operating companies and benefits the environment, and the latter is of more concern to passengers. In addition, metro train delays often occur at busy stations due to delays caused by intermittent passenger crowding, especially during the peak hours. Therefore, the dwell time uncertainty should not be ignored in the timetable optimization problem.

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1.2. Literature review

The literature on train scheduling can be categorized as to the types of schedule involved, namely cyclic or acyclic. Cyclic timetables are mainly applied to passenger railways such as metro, and acyclic timetables are often used for freight railways. For cyclic timetables, Peeters (2003) provided a good overview for the relevant studies before 2003, and discussed their advantages. Kroon et al. (2008) made a stochastic improvement of cyclic timetables by considering the random disturbances in real-world operations. Heydar et al. (2013) developed a mixed integer programming model to minimize both the length of the dispatching cycle and the total dwell time. For acyclic timetables, the early optimization models were summarized in a survey by Cordeau et al. (1998). In 2002, Caprara et al. proposed a graph theoretic formulation for the train timetabling problem using a directed acyclic multigraph. Cacchiani et al. (2010), on the other hand, studied the acyclic train timetabling problem and analyzed the existing optimization models. Because the present research is concerned with cyclic scheduling, the remainder of this review is concerned (except where otherwise stated) with literature on the cyclic case.

In recent years, timetable optimization models have been developed with a variety of criteria, including capacity (Abril et al., 2008), transport demand (Kuo et al., 2010; Canca et al., 2014), robustness (Cacchiani and Toth, 2012; Burdett and Kozan, 2014), delay time (Liebchen et al., 2010; Corman et al., 2012b), overlapping time (Yang et al., 2013), energy consumption (Li and Lo, 2014a,b), passenger waiting time (Niu et al., 2015), and utilization of regenerative energy (Yang et al., 2015a).

Yang et al. (2015a) focused on optimizing the dwell time at each station to improve the utilization of regenerative energy, where the running time and speed profile on each section were considered as constant parameters. With those assumptions, the total tractive energy consumption is also a constant parameter. It may be noted that the main differences between the present research and that of Yang et al. (2015a) are: (a) the decision variables in the present research are speed profiles and running times instead of dwell times; and (b) the present research adopts minimization of travel time as an objective, alongside efficiency of energy utilization.

Because different stakeholders with different interests are involved, it is natural to treat timetable optimization as a multi-objective decision problem. For example, Higgins et al. (1996) proposed a nonlinear mixed-integer programming model to minimize the delay time and the train operation cost, and solved the integer program using a branch-and-bound procedure. Ghoseiri et al. (2004) developed an optimization model to minimize the fuel consumption cost and the total passenger time using the ε-constraint method to find a set of non-dominated solutions that forms the Pareto frontier. Yang et al. (2009) developed an expected value programming model to minimize the delay time and the total passenger time, in which the number of passengers boarding/alighting the train at each station is considered as a fuzzy variable. Corman et al. (2012a) considered the minimization of the consecutive delays between trains and the maximization of the total value of satisfied connections to develop a bi-objective conflict detection and resolution problem.

Li et al. (2013) developed a multi-objective train scheduling model for minimizing the energy, the carbon emission cost and the total passenger time using a fuzzy mathematical programming method to find the optimal solution. Sun et al. (2014) developed a multi-objective optimization model to consider the average travel time, the energy consumption and the user satisfaction and designed a genetic algorithm to solve the model. Yang et al. (2014) developed a bi-objective integer programming model with headway time and dwell time control to increase the utilization of regenerative braking energy and, simultaneously, to shorten the passenger waiting time. On the other hand, Yang et al. (2015b) developed a bi-objective optimization method to determine timetables and speed profiles applying an adaptive genetic algorithm with an optimal train control algorithm to find the optimal solution. Xu et al. (2015) developed a multi-objective timetable optimization model to consider both energy efficiency and service quality for metro systems.

Overall, many studies have considered multiple objectives in the timetable optimization problem, but only a few studies (Li and Yang, 2013; Wu et al., 2015) consider the uncertain factors of the real-world operations in determining the metro timetable. This paper contributes to the current literature by developing a bi-objective programming approach that explicitly considers the dwell time uncertainty in the timetable optimization problem for minimizing both net energy consumption and total travel time. The main contributions of this paper are as follows:

- We consider the uncertain dwell time and formulate the timetable optimization problem as a bi-objective expected value model.
- We use the ε-constraint method within a genetic algorithm framework to obtain the Pareto optimal solutions and determine three optimal timetables suitable for different real-world operation cases.
- We present numerical examples based on the real-world operation data from the Beijing Metro Yizhuang Line to illustrate the practicability of the model as well as the effectiveness of the solution procedure.

1.3. Paper structure

The remainder of the paper is organized as follows. In Section 2, we formulate a bi-objective expected value model to determine the optimal timetable. In Section 3, we design a genetic algorithm combined with the ε-constraint method to solve the developed bi-objective expected value model. In Section 4, two numerical examples based on the real-world data from the Beijing Metro Yizhuang Line are presented. Finally, conclusions are given in Section 5.

2. Model formulation

In this section, we formulate the timetable optimization problem as a bi-objective expected value model to reduce both net energy consumption and total travel time under dwell time uncertainty. Net energy consumption is the difference between the total required tractive energy and the total utilization of regenerative braking energy. Total travel time is the time taken for a train to complete the trip from the starting station to the terminal station.

The metro system considered here is cyclic and regular in operation and linear in configuration, comprising paired tracks and turnaround loops at each end. The analysis is focused on a single track, with traffic in one direction only, which is sufficient in general to provide a complete timetable specification for the cyclic and regular case studied here (Higgins et al., 1996). Every train stops at all stations (i.e., there are no skip-stop or express services). Typically, the headway is a design variable related with passenger demand and train capacity, but it takes a constant value during a fixed operation period such as peak hours or off-peak hours; here, headway is assumed constant for simplified planning purposes. Dwell time is treated as comprising a constant base value plus a non-negative stochastic increment; dwell time and dwell time variability are different at different stations but are constant for all trains at a given station; dwell time is assumed to be no greater than headway; and the same speed profile is assumed for all trains on the same section, but it is different for different sections. Note these assumptions preclude collisions between successive trains, thus making sidings for passing trains unnecessary. Another point worth noting is that a higher total travel time in the line implies a higher cycle time (i.e., the time needed to complete a loop in the line). If the headway is fixed, the number of trains attending the line increases. Consequently, the total energy consumption in the line when trains are following a timetable design with long cycle times also increases. The best solution in terms of energy consumption considering all the line cannot be obtained by minimizing only the energy consumption of one train, because this can produce an increment in the needed fleet size. Therefore, the global solution must balance both total energy consumption and the fleet size in order to achieve the best timetable design. Further assumptions are set out in Section 2.2 below.

2.1. Notations

Notations used throughout the paper are listed as follows and all boldface letters denote the corresponding vectors. All variables are assumed to be integer numbers.

2.3.1. Parameters

i	train index, $i = 1, 2, \ldots, I$
n	station index, $n = 1, 2, \ldots, N$
m	train mass, which is assumed as constant for all trains and at all times
h	time headway, which is assumed as constant throughout the system
$s_{(n,n+1)}$	length of section $(n, n+1)$
$l_{(n,n+1)}$	lower bound of running time on section $(n, n+1)$
$u_{(n,n+1)}$	upper bound of running time on section $(n, n+1)$
t_{1n}	current planned dwell time at station n
t_{2n}	maximum dwell time at station n
y	possible value of dwell time,$y \in [t_{1n}, t_{2n}], y \in \mathbb{Z}$
f_a	maximum tractive force
f_b	maximum braking force
r	basic running resistance
g	additional running resistance caused by gradient and curve
η_1	conversion efficiency of the train traction system (i.e., traction electrical energy from electricity to mechanical energy)
η_2	conversion efficiency of the train braking system (i.e., conversion efficiency from mechanical energy to electrical energy)
β	transmission loss coefficient on regenerative braking energy

2.3.2. Stochastic variables

ξ_n	dwell time at station n

2.3.3. Decision variables

$x_{(n,n+1)}$	running time on section $(n, n+1)$

2.3.4. Intermediate variables

a_{in}	arrival time of train i at station n
d_{in}	departure time of train i from station n
c_{in}	switching time from accelerating phase to coasting phase for train i on section $(n, n+1)$
b_{in}	switching time from coasting phase to braking phase for train i on section $(n, n+1)$
v_{c_n}	speed at switching point from accelerating phase to coasting phase on section $(n, n+1)$
v_{b_n}	speed at switching point from coasting phase to braking phase on section $(n, n+1)$

For simplicity, we denote the decision variables as $\mathbf{x} = \{x_{(n,n+1)} \mid n = 1,2,\ldots,N-1\}$ and the stochastic variables as $\xi = \{\xi_n \mid n = 1,2,\ldots,N\}$. We set the time that the first train arrives at the starting station at time zero (i.e., $a_{11} = 0$). For i $(1 \leqslant i \leqslant I)$, the arrival time and departure time of train i at station n can be specified as follows:

$$a_{in} = \begin{cases} h(i-1), & \text{if } n = 1, \\ h(i-1) + \sum_{k=1}^{n-1}(\xi_k + x_{(k,k+1)}), & \text{if } n = 2,3,\ldots,N. \end{cases} \tag{1}$$

$$d_{in} = \begin{cases} h(i-1) + \xi_1, & \text{if } n = 1, \\ h(i-1) + \sum_{k=1}^{n-1}(\xi_k + x_{(k,k+1)}) + \xi_n, & \text{if } n = 2,3,\ldots,N. \end{cases} \tag{2}$$

Note that $h(i-1)$ in Eqs. (1) and (2) denotes the arrival time of train i at station 1.

2.2. Model assumptions

According to the operation characteristics of metro systems, we formulate the model based on the following assumptions.

(a) Train mass, maximum tractive force, maximum braking force, basic running resistance and additional running resistance are considered as constants.
(b) The regenerative energy is fed back into the overhead contact line and can be immediately utilized to accelerate adjacent trains in the same direction. As shown in Fig. 1a, the regenerative energy from braking train i can be utilized to accelerate trains $i+1$ and $i-1$. If the feedback energy cannot be used immediately, it will be wasted by heating resistors installed on the overhead contact line.
(c) The conversion efficiencies of the train's traction system (from electrical energy to mechanical energy) and braking system (from mechanical energy to electrical energy) are assumed as constants. Based on the assumption b), the regenerative energy is only transmitted between adjacent trains. Therefore, the transmission distance is generally not too long, and the transmission loss coefficient on regenerative energy is also assumed as a constant.
(d) For the busy stations, trains usually have uncertain departure delays. We use t_{1n} and t_{2n} to denote the current planned dwell time and the maximum dwell time with delays, respectively. If station n is a busy station, the probability density function (PDF) of the stochastic dwell time ξ_n is shown in Fig. 2, and formulated as

$$p(\xi_n = y) = \begin{cases} (-2y + 2t_{2n}/[(t_{2n} - t_{1n} + 1)(t_{2n} - t_{1n})]), & \text{if } y \in [t_{1n}, t_{2n}], y \in \mathbb{Z}, \\ 0, & \text{otherwise}. \end{cases} \tag{3}$$

Remark 1. According to the historical data from the Beijing Metro Yizhuang line, we assume the PDF of the dwell time as a triangular distribution. Besides, the stochastic component is nonnegative that is a delay due to heavier passenger traffic than was allowed for in the basic dwell time. Other distributions can also be considered. In our experiments, its impact on the model formulation and solution algorithm is minimal.

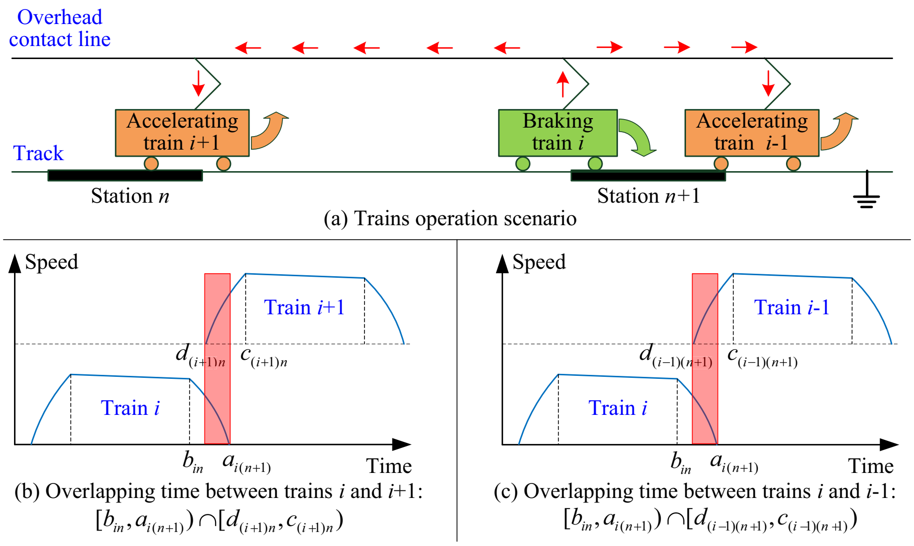


Fig. 1. Utilization of the regenerative energy from braking train i to adjacent trains $i-1$ and $i+1$.

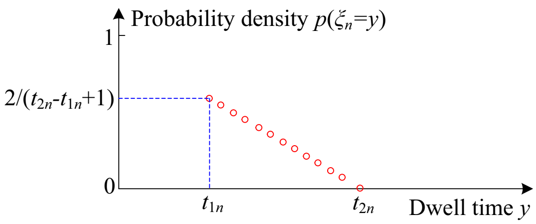


Fig. 2. Probability density function of the stochastic dwell time ξ_n.

2.3. Objective functions

We consider net energy consumption and total travel time as two objective functions in the model. Before we formulate the two objective functions, we first describe the speed profile formulation for a given section.

Based on the optimal train control theory (Howlett, 2000; Howlett et al., 2009; Albrecht et al., 2013), the energy-efficient speed profile for trains within a given section typically consists of four phases: maximum acceleration, cruising, coasting, and maximum braking. The present research, however, is directed at metro networks in which the travel distances are relatively short, with no cruising phase: this leaves three phases, namely maximum acceleration, coasting and maximum braking phases (Howlett and Pudney, 1995; Bocharnikov et al., 2010; Su et al., 2013).

Given a section $(n, n+1)$ and according to the train motion equation, the speed profile for train i is

$$v_{in}(t)=\begin{cases}(f_a-r-g)(t-d_{in})/m, & \text{if } d_{in}\leqslant t<c_{in},\\ v_{c_n}-(r+g)(t-c_{in})/m, & \text{if } c_{in}\leqslant t<b_{in},\\ v_{b_n}-(f_b+r+g)(t-b_{in})/m, & \text{if } b_{in}\leqslant t<a_{i(n+1)},\end{cases} \tag{4}$$

where the first row denotes the maximum accelerating phase, the second row denotes the coasting phase, and the third row denotes the maximum braking phase. Note that $v_{in}(t)$ is determined by decision variables $\mathbf{x}$ and stochastic variables ξ. The switching speeds v_{c_n} and v_{b_n} are determined as follows:

$$\begin{cases}v_{c_n}=(f_a-r-g)(c_{in}-d_{in})/m,\\ v_{b_n}=(f_a-r-g)(c_{in}-d_{in})/m-(r+g)(b_{in}-c_{in})/m.\end{cases} \tag{5}$$

The switching times c_{in} and b_{in} should satisfy the following equations:

$$\begin{cases}v_{b_n}-(f_b+r+g)(a_{i(n+1)}-b_{in})/m=0,\\ mv_{c_n}^2/2(f_a-r-g)+m(v_{c_n}^2-v_{b_n}^2)/2(r+g)+mv_{b_n}^2/2(f_b+r+g)=s_{(n,n+1)},\end{cases} \tag{6}$$

where the first equation denotes that the speed of train i should reduce to zero when it arrives at station $n+1$, and the second equation denotes that the speed profile should satisfy the travel distance constraint. Eqs. (4) and (5) imply that the intermediate variables v_{c_n}, v_{b_n}, c_{in} and b_{in} can be expressed in terms of the arrival time a_{in}, departure time d_{in}, and given parameters.

2.3.1. Total travel time

The total travel time of train i for completing a trip from station 1 to station N is formulated as

$$T_i(\mathbf{x},\xi)=\sum_{n=1}^{N-1}(\xi_n+x_{(n,n+1)}). \tag{7}$$

Total travel time is thus explicitly dependent on both the decision variables and the stochastic variables.

2.3.2. Net energy consumption

Generally speaking, the metro train needs to use energy from the power supply network for increasing the train speed during the accelerating phase, and it regenerates energy during the braking phase. There is no tractive energy consumed or regenerated during the coasting phase or the dwell time period at stations.

Firstly, we calculate the total required tractive energy for train i throughout its whole journey (i.e., the total travel time spent by passengers in the system). For each i $(1\leqslant i\leqslant I)$ and n $(1\leqslant n\leqslant N)$, the required tractive energy for accelerating train i at station n and section $(n, n+1)$ at time t is

$$F_{in}(\mathbf{x},\xi,t)=\begin{cases} f_a v_{in}(\mathbf{x},\xi,t)/\eta_1, & \text{if } d_{in}\leqslant t<c_{in},\\ 0, & \text{if } a_{in}\leqslant t<d_{in}\cup c_{in}\leqslant t<a_{i(n+1)}, \end{cases} \tag{8}$$

where the first condition denotes the accelerating phase on section $(n, n+1)$, and the second condition denotes the dwell time at station n and the coasting and braking phases on section $(n, n+1)$. Note that the tractive energy used here is equivalent to electrical required power in kW · h. Hence, the total required tractive energy for train i throughout its whole journey is

$$J_{iF}(\mathbf{x},\xi)=\sum_{t=d_{i1}}^{a_{iN}}F_{in}(\mathbf{x},\xi,t). \tag{9}$$

Furthermore, we calculate the total utilization of regenerative energy from train i throughout its whole journey. For each i $(1\leqslant i\leqslant I)$ and n $(1\leqslant n\leqslant N)$, the energy regenerated from train i at station n and section $(n, n+1)$ at time t is

$$B_{in}(\mathbf{x},\xi,t)=\begin{cases} 0, & \text{if } a_{in}\leqslant t<d_{in}\cup d_{in}\leqslant t<b_{in},\\ f_b v_{in}(\mathbf{x},\xi,t)\eta_2, & \text{if } b_{in}\leqslant t<a_{i(n+1)}, \end{cases} \tag{10}$$

where the first condition denotes the dwell time at station n and the accelerating and coasting phases on section $(n, n+1)$, and the second condition denotes the braking phase on section $(n, n+1)$. For simplicity, we define

$$Ts=\{[b_{in},a_{i(n+1)})\cap[d_{(i+1)n},c_{(i+1)n})\}\cup\{[b_{in},a_{i(n+1)})\cap[d_{(i-1)(n+1)},c_{(i-1)(n+1)})\}, \tag{11}$$

where Ts denotes the total overlapping time between train i and its adjacent trains $i+1$ and $i-1$; $[b_{in},a_{i(n+1)})\cap[d_{(i+1)n},c_{(i+1)n})$ denotes the overlapping time between braking train i on section $(n, n+1)$ and accelerating train $i+1$ on section $(n, n+1)$ (see Fig. 1b for an illustration); and $[b_{in},a_{i(n+1)})\cap[d_{(i-1)(n+1)},c_{(i-1)(n+1)})$ denotes the overlapping time between braking train i on section $(n, n+1)$ and accelerating train $i-1$ on section $(n+1, n+2)$ (see Fig. 1c for an illustration). Only during the total overlapping time, the regenerative energy from train i can be utilized by trains $i+1$ or $i-1$. Therefore, the total utilization of regenerative energy from train i throughout its whole journey is

$$\begin{aligned} J_{iB}(\mathbf{x},\xi)=&\sum_{n=1}^{N-2}\sum_{t\in Ts}\min\{B_{in}(\mathbf{x},\xi,t)(1-\beta),[F_{(i+1)n}(\mathbf{x},\xi,t)+F_{(i-1)(n+1)}(\mathbf{x},\xi,t)]\}\\ &+\sum_{t\in Ts}\min\{B_{i(N-1)}(\mathbf{x},\xi,t)(1-\beta),F_{(i+1)(N-1)}(\mathbf{x},\xi,t)\}, \end{aligned} \tag{12}$$

which contains two terms: (1) the first term denotes that the utilization of regenerative braking energy of train i from Section 1 to section $(N-2, N-1)$; and (2) the second term denotes that the utilization of regenerative braking energy of train i on section $(N-1, N)$, where train $i-1$ has already arrived at the destination station N, and is therefore not included. In both terms, the minimum operator is used to select between the regenerative braking energy and the tractive energy required to accelerate the train. If the tractive energy is larger than the regenerative braking energy, it means all the regenerative braking energy is being used and additional energy is needed to accelerate the adjacent trains according to the implemented speed profiles. On the other hand, if the tractive energy is smaller than the regenerative energy, it means the regenerative braking energy is more than sufficient to accelerate the adjacent trains, and the remaining regenerative braking energy not used will be wasted by heating resistors installed on the overhead contact lines. To help visualizing the process of computing the utilization of regenerative energy, a pictorial framework is provided in Fig. 3. Finally, the net energy consumption for train i throughout its whole journey is

$$J_i(\mathbf{x}, \xi) = J_{iF}(\mathbf{x}, \xi) - J_{iB}(\mathbf{x}, \xi). \tag{13}$$

The minimum operators in $J_{iB}(\mathbf{x}, \xi)$ make the net energy consumption equation $J_i(\mathbf{x}, \xi)$ neither convex nor continuous (i.e., nonlinear and non-smooth). Therefore, it is difficult to evaluate it using traditional optimization methods.

2.4. Optimization model

In the timetable optimization problem, we want to minimize the total travel time as well as the net energy consumption. Since the two quantities $T_i(\mathbf{x}, \xi)$ and $J_i(\mathbf{x}, \xi)$ to be minimized include stochastic variables, we use the expected value criterion to minimize their average values. Therefore, we formulate the timetable optimization problem as a bi-objective expected value model as follows:

$$\begin{cases} \min & E[T_i(\mathbf{x}, \xi)], E[J_i(\mathbf{x}, \xi)] & \\ s.t. & l_{(n,n+1)} \leqslant x_{(n,n+1)} \leqslant u_{(n,n+1)}, & n = 1, 2, \ldots, N-1, \\ & x_{(n,n+1)} \in \mathbb{Z}, & n = 1, 2, \ldots, N-1, \\ & t_{1n} \leqslant \xi_n \leqslant t_{2n}, & n = 1, 2, \ldots, N-1, \\ & v_{c_n} = (f_a - r - g)(c_{in} - d_{in})/m, & n = 1, 2, \ldots, N-1, \\ & v_{b_n} = (f_a - r - g)(c_{in} - d_{in})/m - (r+g)(b_{in} - c_{in})/m, & n = 1, 2, \ldots, N-1, \\ & v_{b_n} - (f_b + r + g)(a_{i(n+1)} - b_{in})/m = 0, & n = 1, 2, \ldots, N-1, \\ & m v_{c_n}^2/2(f_a - r - g) + m(v_{c_n}^2 - v_{b_n}^2)/2(r+g) + m v_{b_n}^2/2(f_b + r + g) = s_{(n,n+1)}, & n = 1, 2, \ldots, N-1. \end{cases} \tag{14}$$

The first set of constraints ensures that the running time on each section should satisfy its upper and lower bound constraints. The second set of constraints ensures that the decision variables are all integer according to the engineering requirements. The third set of constraints ensures that the stochastic dwell time is valued within an interval. The remaining constraints ensure the proper speed profile for each section.

Solving the bi-objective expected value model presents a number of challenges; in particular: (1) one of the objectives $J_i(\mathbf{x}, \xi)$ in the model is nonconvex and discontinuous (i.e., nonlinear and non-smooth); (2) the model has multiple Pareto optimal solutions (i.e., there does not exist a single solution that simultaneously optimizes both objectives); and (3) it is necessary to repeat a number of times to solve the stochastic model for obtaining the expected value. Therefore, it is difficult to find the optimal solution using the classical optimization methods.

3. Solution algorithm

As explained in Section 2.4, the developed bi-objective expected value model is complicated. This section designs a genetic algorithm (GA) combined with the ε-constraint method to find the Pareto optimal solutions. We apply the ε-constraint method to model (14), to obtain the following modified formulation

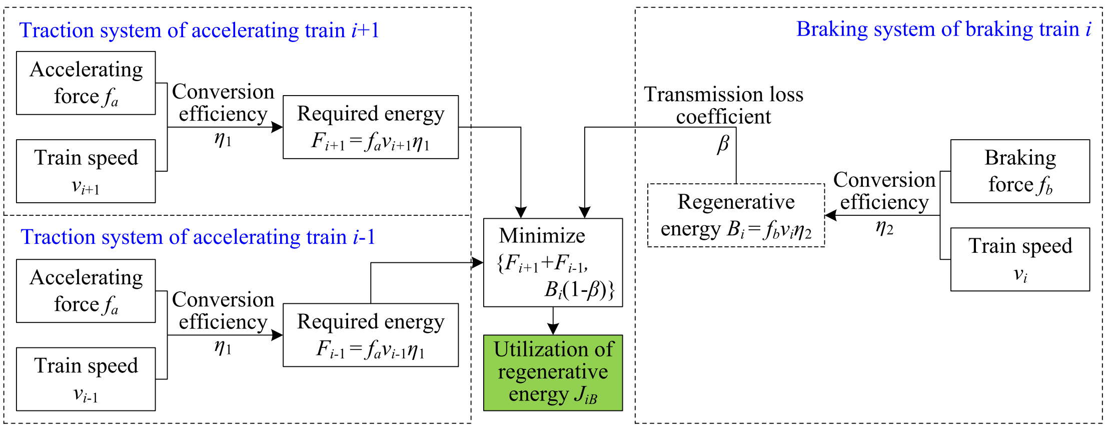


Fig. 3. Computing framework for utilizing the regenerative energy.

$$\begin{cases} \min & E[J_i(\mathbf{x},\xi)] \\ s.t. & E[T_i(\mathbf{x},\xi)] \leqslant \varepsilon, \\ & l_{(n,n+1)} \leqslant x_{(n,n+1)} \leqslant u_{(n,n+1)}, & n = 1,2,\ldots,N-1, \\ & x_{(n,n+1)} \in \mathbb{Z}, & n = 1,2,\ldots,N-1, \\ & t_{1n} \leqslant \xi_n \leqslant t_{2n}, & n = 1,2,\ldots,N-1, \\ & v_{c_n} = (f_a - r - g)(c_{in} - d_{in})/m, & n = 1,2,\ldots,N-1, \\ & v_{b_n} = (f_a - r - g)(c_{in} - d_{in})/m - (r+g)(b_{in} - c_{in})/m, & n = 1,2,\ldots,N-1, \\ & v_{b_n} - (f_b + r + g)(a_{i(n+1)} - b_{in})/m = 0, & n = 1,2,\ldots,N-1, \\ & m v_{c_n}^2/2(f_a - r - g) + m(v_{c_n}^2 - v_{b_n}^2)/2(r+g) + m v_{b_n}^2/2(f_b + r + g) = s_{(n,n+1)}, & n = 1,2,\ldots,N-1, \end{cases} \tag{15}$$

where ε denotes the upper bound of the expected value of T_i. By repeatedly relaxing the value of ε, and re-optimizing the expected value of J_i, the ε-constraint method generates a set of (T_i, J_i) points that form the Pareto frontier.

GA is a stochastic search method firstly initiated by Holland (1975) for solving complex optimization problems. Due to their extensive generality and practical applicability, GAs have been widely applied to different transportation problems, including stochastic network design (Chen and Xu, 2012; Xu et al., 2013), vehicle routing under uncertainty (Allahviranloo et al., 2014), intermodal transportation planning and management (Assadipour et al., 2015), liner hub-and-spoke shipping network design (Zheng et al., 2015), and metro optimization (Niu and Zhou, 2013; Yang et al., 2013, 2014, 2015a, 2015b, 2016b; Xu et al., 2015).

A GA usually starts with an initial set of randomly generated feasible solutions, which are encoded as chromosomes called a population. A new population of chromosomes is generated following the evaluation, selection, crossover and mutation operations. The GA terminates after a given number of iterations of the above steps. The detailed procedure of the genetic algorithm is described below.

3.1. Representation structure

As shown in Fig. 4, we define a chromosome $c = \{c_n \mid n = 1,2,\ldots,N-1\}$ to represent the set of decision variables $\mathbf{x} = \{x_{(n,n+1)} \mid n = 1,2,\ldots,N-1\}$. Each gene c_n in the chromosome represents a decision variable $x_{(n,n+1)}$.

3.2. Initialization

Define an integer number *pop_size* as the population size. For each n $(1 \leqslant n \leqslant N-1)$, randomly initialize an integer number $x_{(n,n+1)} \in [l_{(n,n+1)}, u_{(n,n+1)}]$ as a gene c_n, thus we obtain an initial chromosome $c = (c_1, c_2, \ldots, c_{N-1})$. Check whether this initial chromosome satisfies the ε-constraint $E[T(\mathbf{x},\xi)] \leqslant \varepsilon$. If so, this initial chromosome is a feasible chromosome. Otherwise, delete it and regenerate a new one. In the same way, generate *pop_size* feasible chromosomes as the initial population $\{c_1, c_2, \ldots, c_{pop\_size}\}$.

3.3. Evaluation function

Evaluation function is used to assign a probability p_i of reproduction to each chromosome c_i $(i = 1,2,\ldots,pop\_size)$ so that its likelihood of being selected is proportional to its fitness relative to the other chromosomes in the population. First, we reorder the chromosomes from good to bad according the expected value of the net energy consumption. Then for each $\alpha \in (0,1)$, the evaluation function is defined as $Eval(c_i) = \alpha(1-\alpha)^{i-1}, \; i = 1,2,\ldots,pop\_size$.

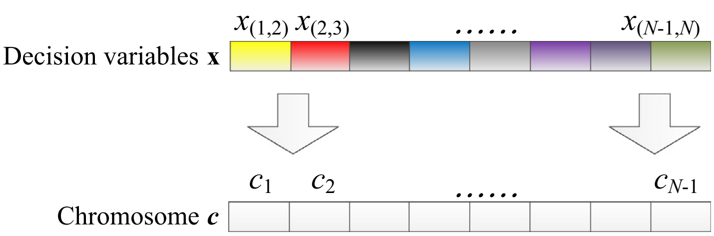


Fig. 4. Structure of a chromosome.

3.4. Selection operation

The selection of chromosomes is done by spinning the roulette wheel which is a fitness-proportional selection. Each time one chromosome is selected for a new child population. Performing this process *pop_size* times, the next generation can be obtained. Define $p_0 = 0$, and calculate the cumulative probability p_i for each chromosome

$$p_i = \sum_{j=1}^{i} Eval(c_j), \quad i = 1, 2, \ldots, pop\_size. \tag{16}$$

The detailed selection procedure is described as follows:

Step 1. Set $j = 1$.
Step 2. Randomly generate a real number $r \in (0, p_{pop\_size}]$.
Step 3. Select the chromosome c_i such that $r \in (p_{i-1}, p_i]$.
Step 4. If $j = pop\_size$, stop; otherwise, set $j = j + 1$ and go to step 2.

3.5. Crossover operation

Define a parameter P_c to denote the probability of crossover operation. Randomly generate a real number $r \in [0, 1]$. If $r < P_c$, select two chromosomes $c_i = (c_i^1, c_i^2, \ldots, c_i^{N-1})$ and $c_{i+1} = (c_{i+1}^1, c_{i+1}^2, \ldots, c_{i+1}^{N-1})$ as the parents. Randomly generate an integer number k from $\{1, 2, \ldots, N-1\}$, thus c_i and c_{i+1} produce two offsprings as $c_x = (c_i^1, \ldots, c_i^k, c_{i+1}^{k+1}, \ldots, c_{i+1}^{N-1})$ and $c_y = (c_{i+1}^1, \ldots, c_{i+1}^k, c_i^{k+1}, \ldots, c_i^{N-1})$ (see Fig. 5). Check whether c_x and c_y are feasible (i.e., satisfying the constraints in model (15)). Take them to replace their parents. If $r \geqslant P_c$, keep c_i and c_{i+1}.

3.6. Mutation operation

Define a parameter P_m to denote the probability of mutation operation. Select a chromosome c_i as the parent for mutation and randomly generate a real number $s \in [0, 1]$. If $s < P_m$, randomly select a gene of the selected chromosome and randomly update this gene within the corresponding upper and lower bounds of the decision variable. Thus, the selected chromosome is also updated. Check whether the new chromosome is feasible (i.e., satisfying the constraints in model (15)). Take it to replace the parental chromosome c_i. If $s \geqslant P_m$, keep c_i.

3.7. General process

A flowchart of the GA-based solution procedure developed in the present research is provided in Fig. 6, and summarized as follows:

Step 1. Initialize parameters: population size *pop_size*, crossover probability p_c, mutation probability p_m, and *max_generation*. Set generation index $i = 1$.
Step 2. Initialize *pop_size* feasible chromosomes as the initial population.
Step 3. Calculate the expected values of the evaluation function for all chromosomes based on the stochastic simulation.
 Step 3.1. Set $U = 0$.
 Step 3.2. Generate the stochastic variable ξ according to the probability density function given in equation (3).
 Step 3.3. For each ξ, calculate the objective function of model (15), and denote as $E(\xi)$;=.
 Step 3.4. Set $U \leftarrow U + E(\xi)$.
 Step 3.5. Repeat step 3.2 to step 3.4 for Y times, where Y is a sufficiently large number.
 Step 3.6. Return U/Y is the expected value of the evaluation function.
Step 4. Select the chromosomes by spinning the roulette wheel.
Step 5. Produce the next generation through the crossover and mutation operations.
Step 6. Check the new generation and ensure it to satisfy the ε-constraint.
Step 7. If $i = max\_generation$, return the best found solution. Otherwise, set $i = i + 1$, and go to step 3.

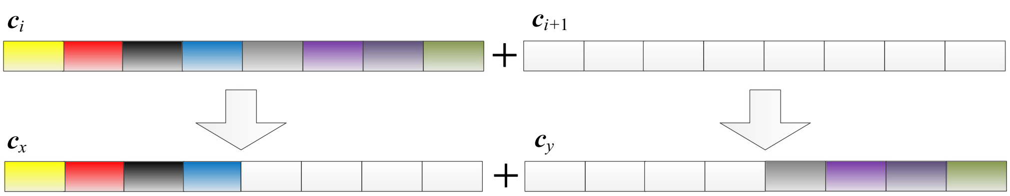


Fig. 5. Crossover operation.

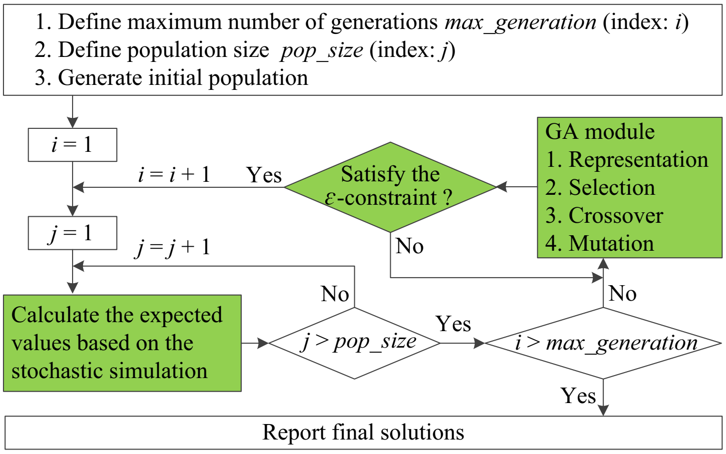


Fig. 6. Flowchart of the GA combined with the ε-constraint solution procedure.

4. Numerical experiments

In this section, two numerical examples based on the real-world operation data from the Beijing Metro Yizhuang Line are presented to illustrate the practicability of the developed model as well as the effectiveness of the solution method. The Yizhuang Line connects the downtown of Beijing and the Yizhuang Economic Development Zone, which covers a length of 22.73 km and consists of 14 stations from Songjiazhuang station to Yizhuang station (see Fig. 7).

We obtained the current operation data of the Beijing Metro Yizhuang Line from the Beijing Mass Transit Railway Operation Corporation Limited (Zhang, 2014). The current planned dwell time at each station, and the current planned running

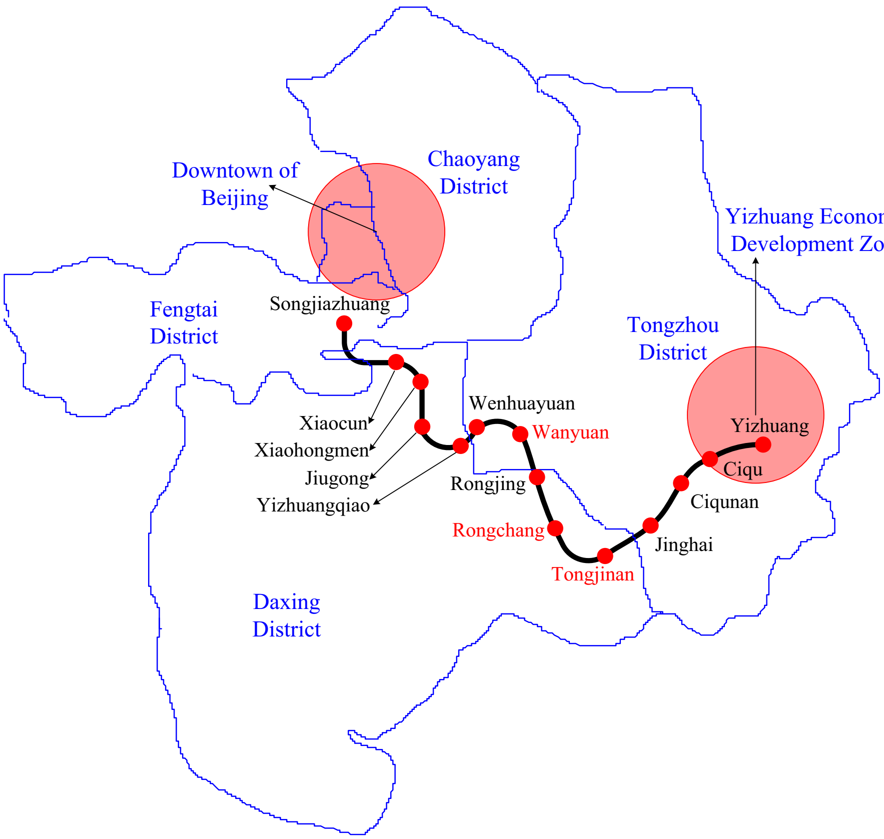


Fig. 7. Illustration of the Beijing Metro Yizhuang Line. *Source:* Zhang, 2014.

time, bounds of running time and length of each section are provided in Table 1. There are three busy stations (i.e., Wenhuayuan, Rongchang and Tongjinan) in the Yizhuang Line that are frequently delayed by over-crowded passengers (Li and Yang, 2013), and the maximum delay is 10 s. Therefore, for station $n = \{6, 9, 10\}$, the PDF of the stochastic dwell time ξ_n is

$$p(\xi_n = y) = \begin{cases} (-y + 40)/55, & \text{if } y = \{30,\ 31,\ \ldots,\ 40\} \\ 0, & \text{otherwise}. \end{cases} \tag{17}$$

The remaining parameters are listed in Table 2.

Based on the provided operation data, the expected values of total travel time and net energy consumption for the current planned timetable are 2086 s and 176.5292 kW·h.

In what follows, Example 1 analyzes the influence of GA parameters on the results to determine their reasonable values. Example 2 provides the Pareto optimal solutions and shows three representative optimal timetables suitable for different real-world operations. Example 3 makes a comparison between the developed bi-objective expected value model and the bi-objective deterministic model (i.e., without considering the dwell time uncertainty). The solution procedure is performed on a personal computer with processor frequency of 2.4 GHz and memory size of 8 GB.

Example 1. This example analyzes the influence of GA parameters (i.e., p_c, p_m, pop_size and max_generation) on the results, such that we can obtain their reasonable values to conduct the following experiments. For model (15), we set $\varepsilon = 2086$ (the expected value of total travel time with the current planned timetable). The results with respect to pop_size and max_generation are shown in Fig. 8. Based on the results, we take pop_size = 60 and max_generation = 100 in the following experiments.

Next, we perform the GA using different combinations of p_c and p_m. The results and the computation time are recorded in Table 3. The 7th result with the minimum best found value and a lower computation time. Therefore, we take $p_c = 0.6$ and $p_m = 0.15$ as the GA parameter values for further experiments.

Example 2. This example provides the Pareto optimal solutions of the developed bi-objective programming approach. By separately maximizing and minimizing the total travel time objective function, we obtain the expected value of maximum total travel time $T_{max} = 2151$ s and the expected value of minimum total travel time $T_{min} = 2021$ s. Thus for model (15), we vary the ε value from 2021 to 2151, and set the interval as 10. By repeatedly performing the solution procedure with pop_size = 60, max_generation = 100, $p_c = 0.6$ and $p_m = 0.15$, we obtain 14 Pareto solutions provided in Table 4 and the Pareto frontier shown in Fig. 9.

The Pareto frontier denotes the set of solutions from which it is impossible to make any reduction on the total energy consumption without increasing the total travel time or vice versa. These obtained Pareto solutions can provide useful guidance to the metro operators for making changes to achieve a balance tradeoff between the two objectives. Fig. 9 shows three representative solutions that are suitable for applying to different real-world operations. We discuss these three solutions as follows.

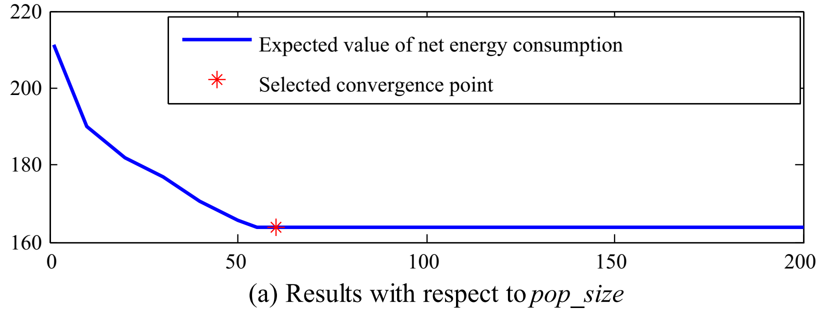
For solution 1, the corresponding optimal timetable is provided in Table 5. The obtained expected values of total travel time and net energy consumption are 2021 s and 212.45 kW·h, respectively. The results show that the expected total travel time of solution 1 is reduced by (2086–2021)/2086 = 3.12% in comparison to the current planned timetable. However, the cost of the expected net energy consumption is increased by (212.45–176.53)/176.53 = 20.35%. From the view point of metro decision-makers, this solution is suitable for operations during peak hours, as it can reduce travel time for more passengers albeit the percentage of travel time reduction is small compared to the percentage of net energy consumption increase.

Table 1
Current planned dwell time at each station, and the current planned running time, bounds of running time and length of each section.

Station	Dwell time (s)	Running time (s)	Lower bound (s)	Upper bound (s)	Length (m)
Songjiazhuang (SJZ)	30	190	185	195	2631
Xiaocun (XC)	30	108	103	113	1275
Xiaohongmen (XHM)	30	157	152	162	2366
Jiugong (JG)	30	135	130	140	1982
Yizhuangqiao (YZQ)	35	90	85	95	993
Wenhuayuan (WHY)	30	114	109	119	1538
Wanyuan (WY)	30	103	98	108	1280
Rongjing (RJ)	30	104	99	109	1354
Rongchang (RC)	30	164	159	169	2338
Tongjinan (TJN)	30	150	145	155	2265
Jinghai (JH)	30	140	135	145	2086
Ciqunan (CQN)	35	102	97	107	1286
Ciqu (CQ)	45	105	100	110	1334
Yizhuang (YZ)	–				

Table 2
Value and unit of some parameters.

Parameter	N	m	h	f_a	f_b	r	g	η_1	η_2	β
Value	14	311,800	90	315,000	258,000	2000	500	0.7	0.8	0.05
Unit	–	kg	s	N	N	N	N	–	–	–



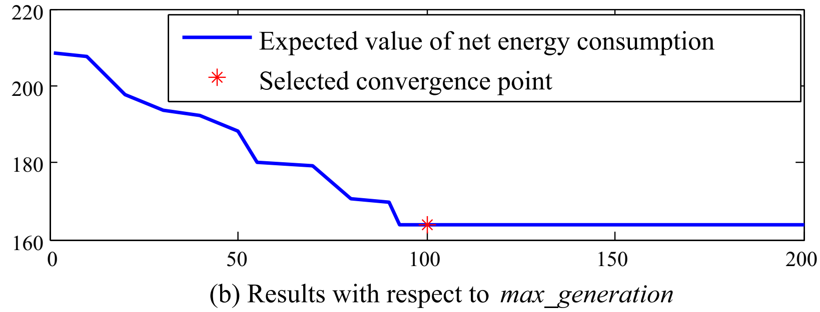


Fig. 8. Results with respect to *pop_size* and *max_generation*.

Table 3
Results with different combinations of p_c and p_m.

No.	p_c	p_m	Best found value	Computation time (s)
1	0.5	0.05	165.62	1760
2	0.5	0.10	165.81	1772
3	0.5	0.15	164.25	1756
4	0.5	0.20	165.82	1751
5	0.6	0.05	165.26	1766
6	0.6	0.10	164.19	1750
7	**0.6**	**0.15**	**164.01**	**1736**
8	0.6	0.20	164.55	1733
9	0.7	0.05	165.09	1749
10	0.7	0.10	165.91	1756
11	0.7	0.15	165.22	1739
12	0.7	0.20	164.31	1755
13	0.8	0.05	165.34	1731
14	0.8	0.10	164.97	1758
15	0.8	0.15	164.28	1746
16	0.8	0.20	164.63	1768

For solution 2, the corresponding optimal timetable is provided in Table 6. The obtained expected values of total travel time and net energy consumption are 2135 s and 156.65 kW·h, respectively. Compared with the current planned timetable, the expected net energy consumption of this solution is reduced by (176.53–156.65)/176.53 = 11.26%, while the expected total travel time of this solution is increased by (2135–2086)/2086 = 2.35%. For some big cities in China with heavy air pollution such as Beijing, the government is vigorously promoting energy saving and emission reduction. Therefore, metro decision-makers may choose this solution for operations during off-peak hours.

For solution 3, the corresponding optimal timetable is provided in Table 7. The obtained expected values of total travel time and net energy consumption are 2071 s and 170.27 kW·h, respectively. The results reveal that the expected values of

Table 4
Pareto optimal results.

No.	ε	Expected total travel time (s)	Expected net energy consumption (kW·h)
1	**2021**	**2021**	**212.45**
2	2031	2031	199.96
3	2041	2041	190.22
4	2051	2051	182.14
5	2061	2061	177.73
6	**2071**	**2071**	**170.27**
7	2081	2081	167.75
8	2091	2091	161.66
9	2101	2101	157.77
10	2111	2111	156.69
11	2121	2121	156.66
12	2131	2131	156.65
13	**2141**	**2135**	**156.65**
14	2151	2135	156.65

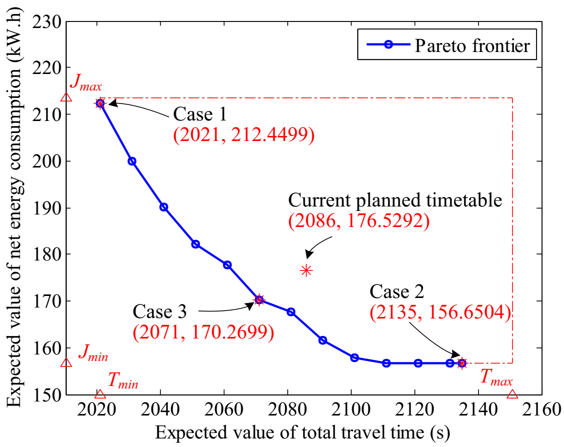


Fig. 9. Pareto frontier.

the total travel time and net energy consumption of the optimal timetable can be reduced by (2086–2071)/2086 = 0.72% and (176.53–170.27)/176.53 = 3.55% in comparison to the current planned timetable. Metro decision-makers can choose this solution when they want a reduction of both total travel time and net energy consumption.

To show the differences among the optimal timetables of the above three solutions and the current planned timetable, we present the four timetables with their expected values of total travel times and net energy consumptions in Fig. 10.

Example 3. This example makes a comparison between the bi-objective deterministic model (BDM, i.e., without considering the dwell time uncertainty) and the developed bi-objective expected value model (BEVM) in this paper.

We assume the dwell time at each station as a deterministic parameter in the modified BEVM (i.e., model (15) modified by the ε-constraint method) to generate the modified BDM as

Table 5
Optimal timetable of solution 1 for the Beijing Metro Yizhuang Line.

Station	SJZ	XC	XHM	JG	YZQ	WHY	WY
Arrival time (s)	0	215	348	530	690	810	949
Departure time (s)	30	245	378	560	725	840	979
Station	RJ	RC	TJN	JH	CQN	CQ	YZ
Arrival time (s)	1077	1206	1395	1570	1735	1867	2012
Departure time (s)	1107	1236	1425	1600	1770	1912	–

Table 6
Optimal timetable of solution 2 for the Beijing Metro Yizhuang Line.

Station	SJZ	XC	XHM	JG	YZQ	WHY	WY
Arrival time (s)	0	224	364	556	725	855	996
Departure time (s)	30	254	394	586	760	885	1026
Station	RJ	RC	TJN	JH	CQN	CQ	YZ
Arrival time (s)	1134	1272	1471	1656	1831	1973	2126
Departure time (s)	1164	1302	1501	1686	1866	2018	–

Table 7
Optimal timetable of solution 3 for the Beijing Metro Yizhuang Line.

Station	SJZ	XC	XHM	JG	YZQ	WHY	WY
Arrival time (s)	0	215	349	535	698	821	962
Departure time (s)	30	245	379	565	733	851	992
Station	RJ	RC	TJN	JH	CQN	CQ	YZ
Arrival time (s)	1096	1230	1422	1602	1771	1911	2062
Departure time (s)	1126	1260	1452	1632	1806	1956	–

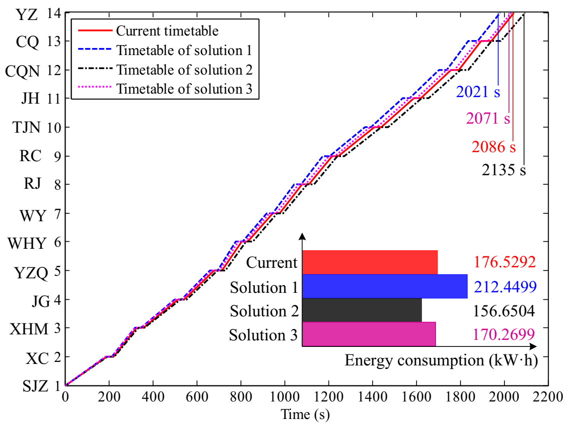


Fig. 10. Optimal timetables of three representative solutions in comparison with the current planned timetable.

$$\begin{cases} \min & J(\mathbf{x}) \\ s.t. & T(\mathbf{x}) \leqslant \varepsilon, \\ & l_{(n,n+1)} \leqslant x_{(n,n+1)} \leqslant u_{(n,n+1)}, \quad n = 1,2,\ldots,N-1, \\ & x_{(n,n+1)} \in \mathbb{Z}, \quad n = 1,2,\ldots,N-1, \\ & v_{c_n} = (f_a - r - g)(c_{in} - d_{in})/m, \quad n = 1,2,\ldots,N-1, \\ & v_{b_n} = (f_a - r - g)(c_{in} - d_{in})/m - (r+g)(b_{in} - c_{in})/m, \quad n = 1,2,\ldots,N-1, \\ & v_{b_n} - (f_b + r + g)(a_{i(n+1)} - b_{in})/m = 0, \quad n = 1,2,\ldots,N-1, \\ & mv_{c_n}^2/2(f_a - r - g) + m(v_{c_n}^2 - v_{b_n}^2)/2(r+g) + mv_{b_n}^2/2(f_b + r + g) = s_{(n,n+1)}, \quad n = 1,2,\ldots,N-1. \end{cases} \tag{18}$$

The maximum and minimum total travel times of the BDM are calculated as 2142 s and 2012 s. By varying the ε value from 2012 to 2142 with an interval of 10, we can obtain 14 Pareto optimal solutions of the BDM. Similarly, we can obtain the expected values of the total travel time and net energy consumption of each Pareto optimal solution as shown in Fig. 11 in comparison with the Pareto frontier of the BEVM.

Fig. 11 shows that the Pareto frontier of the BDM is located in the upper-right in comparison with the Pareto frontier of the BEVM. As we known, the closer the Pareto frontier gets to the lower-left corner (i.e., smaller expected value of net energy consumption and total travel time) the better. Therefore, the results imply that the developed BEVM can achieve a better performance than the BDM.

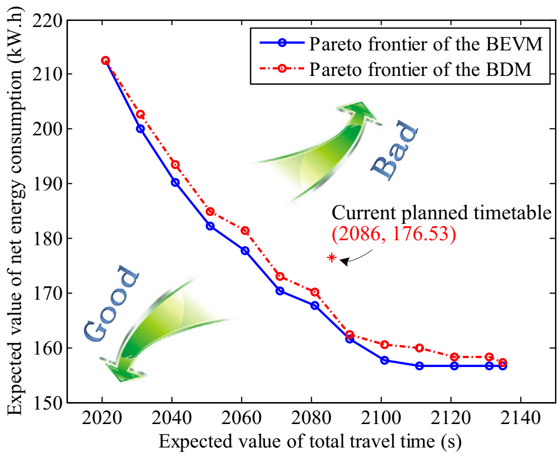


Fig. 11. Comparison between Pareto frontiers of the BEVM and BDM.

5. Conclusion

The main contribution of this paper is to develop a bi-objective programming approach by taking the dwell time uncertainty into consideration to determine the timetable for minimizing both net energy consumption and total travel time. The ε-constraint method combined with the genetic algorithm is designed to find the Pareto optimal solutions. Numerical examples based on the real-world operation data from the Beijing Metro Yizhuang Line are presented. The numerical results provide three representative optimal timetables suitable for real-world operations, which show that the maximum reductions of total travel time and net energy consumption are 3.12% and 11.26%.

In real-world metro systems, there are many other uncertain operating conditions in the timetable optimization problem. For example, the uncertainty of conversion efficiencies between electrical energy and mechanical energy is dependent on the lower and upper limits of voltage in the catenary, the efficiency of vehicle hardware, and so on; the uncertainty of running resistance is influenced by the weather and temperature variations; and the uncertainty of train mass is decided by the dynamic and variable passenger flows. These multiple uncertainties can be studied in future research.

In addition to the travel time and energy consumption, the metro timetable optimization problem may involve other objectives, such as capacity, robustness, delay time, and passenger waiting time. We will extend this work to a more comprehensive set of objective functions to account for different preferences from different stakeholders. Another direction for further research would be to consider variability of passenger demand over the course of a day. This might involve provision for changing timetables (e.g. between peak and off-peak periods), or as an extension of the present model, providing for the minimum planned dwell time to vary between trains or over time.

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