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Hub-and-spoke network design problem under uncertainty considering financial and service issues: A two-phase approach

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ABSTRACT

This paper proposes a bi-objective hub-and-spoke (H&S) network design problem with type-2 (T2) fuzzy transportation cost and travel time described by parametric secondary possibility distributions, which are obtained using three types of mean value (MV) reduction methods. The considered objectives jointly minimize the generalized expectation of the total transportation costs and the maximum travel time requirement in terms of generalized value-at-risk (VaR). To solve the fuzzy bi-objective H&S network design problem, we develop a two-phase approach, where in the first phase we convert the proposed model into its equivalent parametric mixed-integer programming problems by applying an equivalent transformation method. This is followed by the second phase using a fuzzy linear programming approach implemented with an augmented max-min operator to obtain a non-dominated solution that has an equal satisfactory degree on both objectives. Finally, a case study based on the Civil Aeronautics Board (CAB) data set is conducted to demonstrate the effectiveness of the proposed model and solution approach.

1. Introduction

1.1. Background

Hub-and-spoke (H&S) networks have recently received increased attention due to their multiple applications in public transportation, logistics distribution systems, and telecommunications. As a key element of H&S networks, hubs are centralized facilities that serve for consolidating, switching and sorting and allowing for the replacement of direct connections between all nodes with fewer and indirect connections. H&S network design problems are therefore to locate hub facilities and discounted transportation links, allocate origin and destinations nodes (spokes) to hubs, and route flows through the network. From the aspect of operation, the performance of an H&S network can be gauged on different metrics, such as transportation cost, travel time, reliability, flexibility and safety [11]. In this research, the total transportation costs for all origin-destination (O-D) pairs and the maximum travel time requirement between any O-D pair are used as the performance criteria. More specifically, the total transportation costs are related to the benefit of the company, and the maximum travel time requirement is used to evaluate the service quality for customers. It is exactly based on this consideration that this study focuses on the financial and service aspects of H&S networks.

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Designing an H&S network for a company corresponds to a long-term strategic decision, which is typically within an uncertain circumstance occurring when working in a dynamic and chaotic environment. That is, transportation costs, travel times and other parameters cannot always be exactly determined and known in advance. There are several reasons for this, such as lack of transportation information, multiple sources of data, fluctuating nature of parameter values, noise in data, poor statistical analysis, uncertainty in judgment, and so forth. Theoretically, such types of uncertainty can be characterized by type-2 (T2) fuzzy variables by professional judgments or empirical estimates where the membership degree of each point cannot be exactly determined. The three-dimensional nature of a T2 fuzzy variable provides an extra degree of freedom to represent the uncertainty and fuzziness of the real world for application in H&S network design problems. By combining these aforementioned aspects, in this research, we are particularly interested in providing insights into how a company can configure H&S networks to be both efficient (low transportation cost) and effective (high service quality) under uncertainty.

1.2. Literature review

The H&S network design problem is conventionally called the hub location-allocation problem, which primarily consists of the hub median problem and the hub center problem. The study of hub location-allocation was formally proposed by O'Kelly [27,28], who provided a quadratic integer programming formulation for the hub median problem. Skorin-Kapov et al. [35] obtained exact solutions to the hub median problem by developing tight linear relaxations of the formulation. Campbell [4] proposed the first formulation for the hub center problem as a quadratic programming model. Kara and Tansel [12] provided several linear formulations for hub center problems. New MILP formulations of the hub location-allocation problem with fewer variables and constraints were developed by Ernst and Krishnamoorthy [9]. For a detailed review of the hub location-allocation problem and its variations, please see Alumur and Kara [1] and Farahani et al. [10].

The purpose of this paper is to study the hub location-allocation problems under uncertainty, which is an active research area in the literature. The significance of uncertainty has motivated some researchers to address hub location problems with random parameters. For example, Marianov and Serra [22] focused on stochasticity at the hub nodes by representing hub airports as M/D/c queues and limiting the number of airplanes that can queue at an airport through chance constraints. Yang [37] presented a two-stage stochastic programming model for air freight hub location and flight route planning under seasonal demand variations. Contreras et al. [8] studied the stochastic uncapacitated hub location problem in which uncertainty is associated with demands and transportation costs. Sim et al. [34] attempted to address hub location-allocation with stochastic time and utilized a chance-constrained formulation to model the minimum service-level requirement. Mohammadi et al. [25] proposed a new stochastic multi-objective multi-mode transportation model for the hub location-allocation problem under uncertainty. Moreover, some new methods have also been developed to model hub location-allocation problems under possibilistic uncertainty. For instance, Taghipourian et al. [36] presented a fuzzy integer linear programming approach for the dynamic virtual hub location problem with the aim of minimizing the transportation cost in a network. Chou [7] proposed a fuzzy multiple criteria decision-making model for evaluating and selecting the container transshipment hub port. Yang et al. [38], 39] presented two classes of hub location-allocation problems with fuzzy travel times based on the credibility criterion. Mohammadi and Moghaddama [26] proposed a bi-objective fuzzy hub location-allocation problem with the choice of a transportation mode over inter-hub links by incorporating a fuzzy M/M/1 queuing system.

In the fuzzy community, it is well known that Zadeh [41] first proposed the concept of a type-2 (T2) fuzzy set as an extension of the ordinary fuzzy set. A T2 fuzzy set is characterized by a fuzzy membership function, where the degree of membership for any element in this set is a fuzzy number in the interval [0, 1]. Since then, T2 fuzzy set theory has been well studied in the literature [5,6,16,23,24,30,33]. Among them, Liang and Mendel [16] proposed the concept of interval T2 fuzzy sets, which can address the operations via interval arithmetics; Mendel and John [23] noted that a T2 fuzzy set represents the uncertainty in terms of a secondary membership function and footprint of uncertainty; Mitchell [24] used the concept of embedded type-1 (T1) fuzzy numbers to provide a method for ranking T2 fuzzy numbers; Chen and Chang [6] proposed a new method for fuzzy rule interpolation for sparse fuzzy rule-based systems with the ratio of fuzziness of interval T2 fuzzy sets; and Qin et al. [33] extended the VIKOR method based on prospect theory for multiple attribute decision making under an interval T2 fuzzy environment. In fuzzy possibility theory [20], a T2 fuzzy variable is a variable-based approach for handling T2 fuzziness, and it is characterized by a T2 possibility distribution function with a three-dimensional structure. The possibility that a T2 fuzzy variable takes its value is a regular fuzzy variable (RFV), which is easier to be determined than a crisp number in practical applications. Therefore, a T2 fuzzy variable is a more advisable tool for characterizing fuzziness than an ordinary fuzzy variable. To reduce uncertainty in the secondary membership function, Karnik and Mendel [13] proposed a defuzzification method with the concept of centroid of a T2 fuzzy set; Liu [19] proposed a centroid-type reduction strategy for a general T2 fuzzy logic system; and Qin et al. [31], 32] developed the critical value (CV) and mean value (MV) reduction methods based on nonlinear fuzzy integrals. Based on the possibility measure, Yang et al. [40] reduced the uncertainty embedded in the secondary possibility distribution of a T2 fuzzy variable by a fuzzy integral, and Bai and Liu [3] developed VaR-based reduction methods for T2 fuzzy variables.

A number of conclusions from the survey of the literature review can be drawn, including the following:

- Most formulations for the H&S network design problem consider transportation costs and travel times separately in the literature.
- The number of research works that address T2 fuzzy uncertainty in the H&S network design problem is fairly small.

- The proposed methodologies in the literature neglect the multi-criteria decision making nature of the H&S network design problem.
- Most of the optimization models developed in the literature focus on a single objective. Only a few studies address multiple objectives.

1.3. The focus and proposed methods

With the above conclusions, this paper intends to develop a bi-objective H&S network design problem, in which the T2 fuzzy transportation costs and travel times are characterized by parametric secondary possibility distributions. We first employ MV reduction methods to reduce the uncertainty in the secondary possibility distributions of uncertain transportation costs and travel times. Subsequently, we formulate two major objectives based on the generalized credibility measure, which is the main novelty of this study compared to previous works. The major new contributions are summarized as follows.

(i) We propose a new credibilistic parametric method to characterize T2 fuzzy transportation costs and travel times in the H&S network design problems by using variable possibility distributions rather than fixed possibility distributions. The parametric possibility distributions are obtained by using MV reduction methods to the secondary possibility distributions of T2 fuzzy transportation costs and travel times and are characterized by two types of parameters. The parameters describe the degree of uncertainty of fuzzy parameters such that the transportation cost and travel time information cannot be lost by MV reduction methods.
(ii) We develop a bi-objective H&S network design model based on the generalized credibility measure. More specifically, the first objective is to minimize the total transportation costs by using the generalized expectation. The second objective is to minimize the maximum travel time requirement in terms of generalized value-at-risk (VaR). When the T2 fuzzy transportation costs and travel times are mutually independent (i.e., the joint possibility distribution of T2 fuzzy transportation costs and travel times can be represented as the minimum of their marginal possibility distributions), we discuss the equivalent representations of the generalized expected objective and credibility constraints.
(iii) We design a two-phase approach for solving the bi-objective H&S network design problem. In the first phase, we transform the proposed model into its equivalent parametric mixed-integer programming problems by applying an equivalent transformation method. In the second phase, we transform the equivalent multi-objective parametric programming model into a single-objective model by using a fuzzy linear programming approach implemented with an augmented max-min operator to obtain a non-dominated solution that has an equal satisfactory degree on both objectives.
(iv) We provide a case study based on the CAB data set to demonstrate the effectiveness of the proposed model and optimization method. Compared to its counterpart in a T1 fuzzy environment, the computational results reported in the numerical experiments demonstrate the superiority of the proposed approach. Further analysis shows that decision makers can select distinct values of parameters depending on their preferences in our optimization method.

The remainder of this paper is organized as follows. In Section 2, we introduce some fundamental knowledge with respect to T2 fuzzy variables. In Section 3, we develop a fuzzy bi-objective H&S network design problem based on the generalized credibility measure. In Section 4, we derive the equivalent representations of the generalized expected objective and credibility constraints. Section 5 develops a two-phase approach for solving the proposed model. Section 6 implements some numerical experiments. Conclusions and future research directions are discussed in Section 7.

2. Preliminaries

For the completeness of this paper, we shall introduce some fundamental knowledge with regard to T2 fuzzy variables, the reduction methods for T2 fuzzy variables and the generalized credibility measure in this section.

2.1. T2 fuzzy variable

As an extension of the T1 fuzzy variable, the T2 fuzzy variable aims to fuzzify membership values, where the membership value is no longer an exact value but rather a T1 regular fuzzy variable (RFV) on [0, 1]. The following discussion focuses on the introduction of T2 fuzzy variables proposed by Liu and Liu [20].

Definition 1 [20]**.** Let $\mathcal{A}$ be an ample field on the universe Γ and $\tilde{\text{P}}\text{os} : \mathcal{A} \longmapsto \mathcal{R}([0,1])$ be a set function on $\mathcal{A}$ such that $\{\tilde{\text{P}}\text{os}(A) | \mathcal{A} \ni A \ atom\}$ is a family of mutually independent RFVs. We call $\tilde{\text{P}}\text{os}$ a fuzzy possibility measure if it satisfies the following conditions:

(i) $\tilde{\text{P}}\text{os}(\emptyset) = 0$;
(ii) For any subclass $\{A_i | i \in I\}$ of $\mathcal{A}$ (finite, countable or uncountable),

$$\tilde{\text{P}}\text{os}\left(\bigcup_{i \in I} A_i\right) = \sup_{i \in I} \tilde{\text{P}}\text{os}(A_i).$$

Moreover, if $\mu_{\tilde{\text{Pos}}(\Gamma)}(1)=1$, then we call $\tilde{\text{Pos}}$ a regular fuzzy possibility measure.

Definition 2 [20]**.** Let $(\Gamma, \mathcal{A}, \tilde{\text{Pos}})$ be a fuzzy possibility space (FPS). A map $\tilde{\xi}=(\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_m): \Gamma \longmapsto \Re^m$ is called an m-ary T2 fuzzy vector if for any $r=(r_1, r_2, \ldots, r_m) \in \Re^m$, the set $\{\gamma \in \Gamma | \tilde{\xi}(\gamma) \leq r\}$ is an element of A, i.e., $\{\gamma \in \Gamma | \tilde{\xi}(\gamma) \leq r\} = \{\gamma \in \Gamma | \tilde{\xi}_1(\gamma) \leq r_1, \tilde{\xi}_2(\gamma) \leq r_2, \ldots, \tilde{\xi}_m(\gamma) \leq r_m\} \in A$. When $m=1$, the map $\tilde{\xi}: \Gamma \longmapsto \mathcal{R}$ is called a T2 fuzzy variable.

Definition 3 [20]**.** Let $\tilde{\xi}=(\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_m)$ be a T2 fuzzy vector defined on an FPS $(\Gamma, \mathcal{A}, \tilde{\text{Pos}})$. The secondary possibility distribution function of $\tilde{\xi}$, denoted by $\tilde{\mu}_{\tilde{\xi}}(x)$, is a map $\Re^m \longmapsto \mathcal{R}[0,1]$ such that

$$\tilde{\mu}_{\tilde{\xi}}(x) = \tilde{\text{Pos}}\{\gamma \in \Gamma | \tilde{\xi}(\gamma) = x\}, x \in \Re^m.$$

The T2 normal fuzzy variable is an important concept proposed by Qin et al. [32], which has a secondary possibility distribution as shown below.

Definition 4 [32]**.** A T2 fuzzy variable $\tilde{\eta}$ is called normal if its secondary possibility distribution $\tilde{\mu}_{\tilde{\eta}}(x)$ is

$$\left(\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) - \theta_l \min\left\{1-\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)\right\}, \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) + \theta_r \min\left\{1-\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)\right\}\right)$$

for any $x \in \Re$, where $\mu \in \Re$, $\sigma > 0$, and $\theta_l, \theta_r \in [0,1]$ are two parameters characterizing the degree of uncertainty that $\tilde{\eta}$ takes the value x.

Motivated by the concept of T2 normal fuzzy variables, we specifically extend the definition of the secondary possibility distributions for asymmetric T2 normal fuzzy variables in the following:

Definition 5. A T2 fuzzy variable $\tilde{\xi}$ is called asymmetric normal if its secondary possibility distribution $\tilde{\mu}_{\tilde{\xi}}(x)$ is

$$\left(\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) - \theta_l \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) + \theta_r\left(1-\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)\right)\right)$$

for any $x \in \Re$, where $\mu \in \Re$, $\sigma > 0$, and $\theta_l, \theta_r \in [0,1]$ are two parameters characterizing the degree of uncertainty that $\tilde{\xi}$ takes the value x. For simplicity, the asymmetric T2 normal fuzzy variable $\tilde{\xi}$ with the above distribution is denoted by $\tilde{n}(\mu, \sigma^2; \theta_l, \theta_r)$.

In comparison to the conventional T2 fuzzy variable [20], the novelties of this study include three main aspects. First, to make it easier and more effective access of T2 fuzzy variable, this paper extends the definition of the secondary possibility distributions for asymmetric T2 fuzzy variables. Second, under special circumstances, this paper proposes the analytical expression and reasoning rule for the asymmetric T2 normal fuzzy variable. Third, this asymmetric description can easily be applied in practical situations with its simple and direct form. By considering these three aspects, we take the first initiative to investigate the asymmetric T2 fuzzy variable from a more realistic viewpoint, which contributes a new perspective to T2 fuzzy theories.

2.2. Reduction methods for T2 fuzzy variable

Due to the fuzzy membership functions, a T2 fuzzy variable should be defuzzified before being applied to practical problems. For this purpose, some defuzzification methods have been presented in the literature, such as Karnik and Mendel [13] and Liu et al. [21]. In this section, we introduce three types of mean reduction methods for a T2 fuzzy variable proposed by Qin et al. [32]. Compared with the existing methods in the literature, the proposed methods are easy to use in formulating the model with T2 fuzzy coefficients.

To define the reduction methods for T2 fuzzy variables, we first introduce three types of mean values for an RFV by adopting the criteria proposed by Liu and Liu [17], including the upper expected value criterion, lower expected value criterion and expected value criterion.

Definition 6 [17]**.** Let ξ be an RFV. The upper expected value, i.e., $E^*[\xi]$, of ξ is defined by

$$E^*[\xi] = \int_0^{+\infty} \text{Pos}\{\xi \geq r\}\text{dr} - \int_{-\infty}^0 \text{Nec}\{\xi \leq r\}\text{dr},$$

while the lower expected value, i.e., $E_*[\xi]$, of ξ is defined by

$$E_*[\xi] = \int_0^{+\infty} \text{Nec}\{\xi \geq r\}\text{dr} - \int_{-\infty}^0 \text{Pos}\{\xi \leq r\}\text{dr}.$$

The expected value, i.e., $E[\xi]$, of ξ is defined by

$$E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geq r\}\mathrm{d}r - \int_{-\infty}^{0} \mathrm{Cr}\{\xi \leq r\}\mathrm{d}r.$$

According to the definitions of the expected value of RFV, if $\xi = (r_1, r_2, r_3)$ is a regular triangular fuzzy variable, then its three expected values are calculated by the following formulas:

$$E_*[\xi] = \frac{r_1 + r_2}{2}, E[\xi] = \frac{r_1 + 2r_2 + r_3}{4}, E^*[\xi] = \frac{r_2 + r_3}{2}.$$

The following theorem introduces the MV reduction methods for the T2 normal fuzzy variable.

Theorem 1 [32]**.** *Let $\tilde{\eta}$ be a T2 normal fuzzy variable defined as $\tilde{n}(\mu, \sigma^2; \theta_l, \theta_r)$. Then, we have the following:*

(i) *With the E_* reduction method, the reduction η_1 of $\tilde{\eta}$ has the following parametric possibility distribution*

$$\mu_{\eta_1}(x) = \begin{cases} \frac{(2-\theta_l)\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{2}, & \text{if } x \leq \mu - \sigma\sqrt{2\ln 2} \text{ or } x \geq \mu + \sigma\sqrt{2\ln 2} \\ \frac{(2+\theta_l)\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)-\theta_l}{2}, & \text{if } \mu - \sigma\sqrt{2\ln 2} < x < \mu + \sigma\sqrt{2\ln 2}. \end{cases}$$

(ii) *With the E reduction method, the reduction η_2 of $\tilde{\eta}$ has the following parametric possibility distribution*

$$\mu_{\eta_2}(x) = \begin{cases} \frac{(4+\theta_r-\theta_l)\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{4}, & \text{if } x \leq \mu - \sigma\sqrt{2\ln 2} \text{ or } x \geq \mu + \sigma\sqrt{2\ln 2} \\ \frac{(4-\theta_r+\theta_l)\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)+\theta_r-\theta_l}{4}, & \text{if } \mu - \sigma\sqrt{2\ln 2} < x < \mu + \sigma\sqrt{2\ln 2}. \end{cases}$$

(iii) *With the E^* reduction method, the reduction η_3 of $\tilde{\eta}$ has the following parametric possibility distribution*

$$\mu_{\eta_3}(x) = \begin{cases} \frac{(2+\theta_r)\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{2}, & \text{if } x \leq \mu - \sigma\sqrt{2\ln 2} \text{ or } x \geq \mu + \sigma\sqrt{2\ln 2} \\ \frac{(2-\theta_r)\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)+\theta_r}{2}, & \text{if } \mu - \sigma\sqrt{2\ln 2} < x < \mu + \sigma\sqrt{2\ln 2}. \end{cases}$$

In the following, similar to the proof of Theorem 1, we can deduce the MV reduction methods for the asymmetric T2 normal fuzzy variable.

Theorem 2. *Let $\tilde{\xi}$ be an asymmetric T2 normal fuzzy variable defined as $\tilde{n}(\mu, \sigma^2; \theta_l, \theta_r)$. Then, we have the following:*

(i) *Using the E_* reduction method, the reduction ξ_1 of $\tilde{\xi}$ has the following parametric possibility distribution*

$$\mu_{\xi_1}(x) = \frac{(2-\theta_l)}{2}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

(ii) *Using the E reduction method, the reduction ξ_2 of $\tilde{\xi}$ has the following parametric possibility distribution*

$$\mu_{\xi_2}(x) = \frac{(4-\theta_l-\theta_r)}{4}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) + \frac{\theta_r}{4}.$$

(iii) *Using the E^* reduction method, the reduction ξ_3 of $\tilde{\xi}$ has the following parametric possibility distribution*

$$\mu_{\xi_3}(x) = \frac{(2-\theta_r)}{2}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) + \frac{\theta_r}{2}.$$

Theorem 2 shows that the reduced fuzzy variables ξ_1, ξ_2 and ξ_3 are characterized by parametric possibility distributions, i.e., their possibility distributions depend on the parameters θ_l and θ_r. Because the parameters θ_l and θ_r vary in the unit interval [0, 1], the parametric possibility distributions run over the entire footprints of T2 fuzzy variables, which are plotted in Fig. 1. That is, the proposed method reduces uncertain information embedded in the secondary possibility distribution, and it retains the most important information in its parametric possibility distributions.

2.3. Generalized credibility measure

The reduced fuzzy variables obtained using the E^*, E_* and E reduction methods are not always normalized, which are shown in Fig. 1. Hence, it is necessary to generalize the concept of credibility measure. Suppose that ξ is a reduced fuzzy variable with a parametric possibility distribution $\mu_\xi(x;\ \theta_l,\ \theta_r)$. The generalized credibility [32] of fuzzy event $\{\xi \leq r\}$ is computed by

$$\tilde{\mathrm{C}}\mathrm{r}\{\xi \leq x\} = \frac{1}{2}\left(\sup_{t\in\Re}\mu(t;\tilde{\theta}) + \sup_{t\leq x}\mu(t;\tilde{\theta}) - \sup_{t>x}\mu(t;\tilde{\theta})\right), t \in \Re.$$

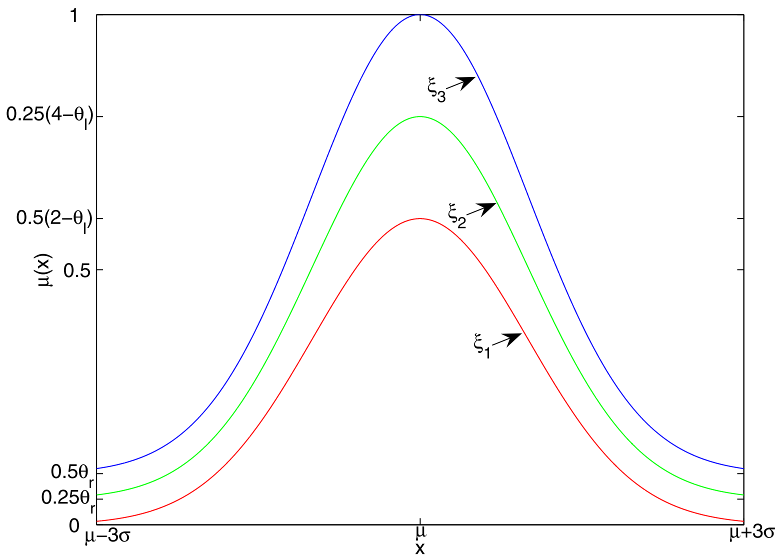

Fig. 1. The parametric possibility distributions of reduced fuzzy variables ξ_1, ξ_2 and ξ_3.

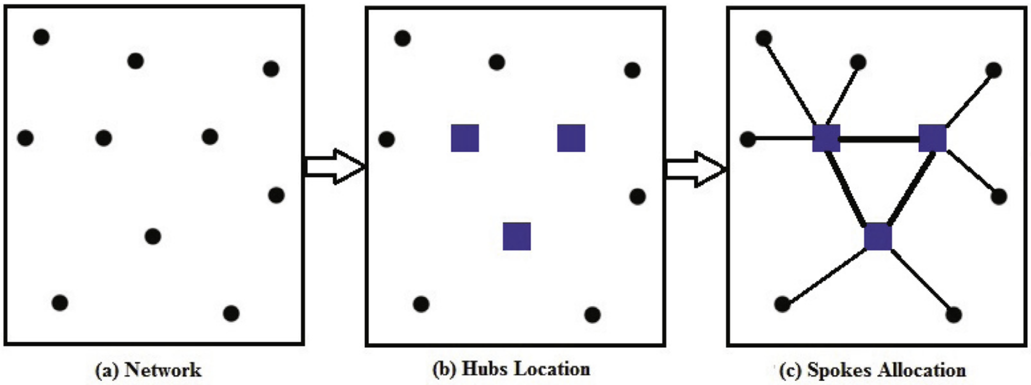

Fig. 2. An example of H&S network.

Based on the generalized credibility measure, the generalized expectation [32] is defined as follows:

$$\tilde{E}[\xi] = \int_0^{+\infty} \tilde{\mathrm{C}}\mathrm{r}\{\xi \geq x\}\mathrm{d}x - \int_{-\infty}^{0} \tilde{\mathrm{C}}\mathrm{r}\{\xi \leq x\}\mathrm{d}x,$$

provided that at least one of the two integrals is finite.

For any given $\alpha \in (0, 1)$, the generalized VaR [40] is computed by

$$\xi_{\mathrm{VaR}}(\alpha) = \min\{r \mid \tilde{\mathrm{C}}\mathrm{r}\{\xi \leq r\} \geq \alpha\}.$$

3. The hub-and-spoke network design problem

The H&S network design problem aims to determine which nodes are located as hubs (hub locations) and spokes are allocated to the located hubs (spoke allocations) such that every flow is first routed through one or two hubs before being disseminated to its destination in a network of nodes in which each pair has a given flow. An example of the H&S network is illustrated in Fig. 2. At the H&S networks, a large set of origin nodes are connected by some intermediate nodes known as hubs to a set of destination nodes. Therefore, H&S networks provide a situation to use less connection links between the nodes. By using fewer numbers of transportation links, this approach concentrates on flows and reduces the transportation cost and travel time in the network since it employs the economies of scale in the links between hubs while hub nodes perform the switching and flow distribution.

Designing the H&S network is a long-term strategic planning problem that can be affected by uncertainties. That is, transportation costs and travel times may change after location decisions have been made. On the one hand, transportation costs can be adjusted from day to day or week by week, which indicates that the transportation cost might be variable in a period because of fuel price fluctuations. On the other hand, with variability in the travel time from an origin to a destination, there is a possibility of not delivering a flow on time. A failure in on-time delivery may result in considerable and non-measurable loss, such as lost-opportunity and lost-sale costs due to causing unsatisfied customers. With these concerns, we study the H&S network design problem with uncertain transportation costs and travel times.

3.1. Motivations for considering T2 fuzzy environment

It is widely recognized that uncertain features always exist in the H&S network, as it is often shown in the transportation process between the O-D pairs. In general, an H&S network design plan should be drawn up before the hub facilities are located in the network. That is, the transportation costs and travel times between the O-D pairs cannot be determined in advance, leading to the inherent uncertainty, which is the motivation for considering the T2 fuzzy environment in this study.

Practically, there are several reasons that cause these uncertainties, such as lack of transportation information, multiple sources of data, fluctuating nature of parameter values, noise in data, poor statistical analysis, uncertainty in judgment, and so forth. Generally, possible values of parameters can be provided by experts in linguistic terms, in which each value may have a different importance or possibility. Such types of linguistic information can be expressed by T2 fuzzy variables, where the membership degree of each point cannot be exactly determined. This three-dimensional nature of a T2 fuzzy variable provides an extra degree of freedom to represent uncertainty for applying in fuzzy H&S network design problems.

With the above analyses, this paper aims to develop a fuzzy bi-objective H&S network design problem with T2 fuzzy transportation costs and travel times characterized by flexible parametric secondary possibility distributions that can be easily determined by experts' experience in practice. To manage the computational complexity, we will employ parametric possibility distributions rather than fixed possibility distributions to describe the reduced transportation costs and travel times, and the parametric possibility distributions are obtained by using the MV reduction methods for T2 fuzzy transportation costs and travel times. In other words, the reduced transportation costs and travel times have parametric possibility distributions such that they can serve as the representatives of T2 fuzzy transportation costs and travel times. When the parameters vary in the unit interval [0, 1], the distribution functions run over the entire footprints of T2 fuzzy transportation costs and travel times. In the following, we shall adopt this modeling idea to formulate a bi-objective fuzzy H&S network design problem.

3.2. Formulation of problem

In this section, we develop a mathematical formulation for the bi-objective H&S network design problem under uncertainty. This model accounts for the uncertainty in transportation costs and travel times when designing the H&S network such that the total transportation costs and the maximum travel time requirement between any OD pair are simultaneously minimized. The problem is modeled using the following notation and mathematical formulation.

Indices and sets:

N: the set of all nodes, indexed by i, j, k, m; $N = \{1, 2, \ldots, n\}$;

i, j: the spoke nodes;

k, m: the hub nodes.

Parameters:

d_1: the cost discount factor on links between hubs, $0 < d_1 \leq 1$;

d_2: the time discount factor on links between hubs, $0 < d_2 \leq 1$;

α: the predetermined generalized credibility level, $0 < \alpha \leq 1$;

p: the number of hubs to be selected;

R: the maximum travel time requirement;

W_{ij}: the demand from node i to node j;

$\widetilde{C}_{ij}$: the T2 fuzzy flow shipment cost between nodes i and j;

$\widetilde{F}_k$: the T2 fuzzy set-up cost for establishing a hub at node k;

$\widetilde{T}_{ij}$: the T2 fuzzy travel time between nodes i and j accordingly.

Decision variables:

$Z_{ik} = 1$ if spoke i is allocated to hub k; $Z_{ik} = 0$ otherwise. In particular, $Z_{kk} = 1$, which indicates that node k serves as a hub.

$X_{ikmj} = 1$ if there exists a path from node i to j via hub k first then m; $X_{ikmj} = 0$ otherwise.

Assumptions:

(A1) The hubs are fully interconnected;

(A2) There are no capacities involved;

(A3) Each non-hub node is allocated to exactly one hub (i.e., single-allocation);

(A4) Direct transportation between non-hub nodes is not allowed;

(A5) The number of hubs to be located is predetermined.

Remark 1. Assumption (A5) is very common and basic in the study of H&S network design problems, which has been used in Campbell [4] and Kara and Tansel [12]. It can be observed that the number of hubs to locate is defined exogenously and denoted by p. Since the H&S network design problem under consideration is of strategic planning type and valid for a long time horizon, in which the number of hubs can generally be controlled by the company, hence it is predetermined. Furthermore, the company can analyze the H&S network topology by varying the number of hubs in this modeling framework. To handle this situation, we assume that the number of hubs that must be located is pre-specified in this paper.

Objective Functions: In this study, two objectives will be taken into consideration, that is, the total transportation costs as the financial objective and the maximum travel time requirement as the service objective. For simplicity, we denote a decision vector as $(\mathbf{X}, \mathbf{Z}) = \{(X_{ikmj}, Z_{ik}) | \forall i, j, k, m \in N\}$.

(1) The total transportation costs for all O-D pairs is represented as

$$\sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\widetilde{C}_{ik} + d_1\widetilde{C}_{km} + \widetilde{C}_{mj})X_{ikmj} + \sum_{k\in N}\widetilde{F}_k Z_{kk},$$

which is the sum of the total flow shipment costs and set-up costs for locating the hubs.

Based on the generalized expectation criterion, the financial objective function can be formulated as

$$\mathrm{C}(\mathbf{X}, \mathbf{Z}) = \widetilde{E}\left[\sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\widetilde{C}_{ik} + d_1\widetilde{C}_{km} + \widetilde{C}_{mj})X_{ikmj} + \sum_{k\in N}\widetilde{F}_k Z_{kk}\right]. \tag{1}$$

(2) The travel time on a path $i\rightarrow k\rightarrow m\rightarrow j$ between O-D pairs is represented as

$$\left(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj}\right)X_{ikmj}, \forall i, j, k, m \in N.$$

According to the generalized VaR criterion, the service objective aims to minimize the generalized VaR of the maximum travel time requirement in the sense that

$$\min\left\{R \middle| \tilde{\mathrm{C}}\mathrm{r}\left\{(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj})X_{ikmj} \le R\right\} \ge \alpha, \forall i, j, k, m \in N\right\}, \tag{2}$$

which is typically equivalent to the following representation:

$$\min\ \mathrm{T}(\mathbf{X}, \mathbf{Z}) = R \text{ such that } \tilde{\mathrm{C}}\mathrm{r}\left\{(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj})X_{ikmj} \le R\right\} \ge \alpha, \forall i, j, k, m \in N.$$

Constrains:

Eq. (3) requires that exactly p hubs should be located in the H&S network,

$$\sum_{k\in N} Z_{kk} = p. \tag{3}$$

Eq. (4) states that spoke i can be allocated to hub k only if hub k has been open,

$$Z_{ik} \le Z_{kk}. \tag{4}$$

Eq. (5) imposes a single assignment of nodes to hubs,

$$\sum_{k\in N} Z_{ik} = 1. \tag{5}$$

Eq. (6) ensures that path $i\rightarrow k\rightarrow m\rightarrow j$ is a valid path in the network if and only if spokes i and j are assigned to hubs k and m, respectively,

$$X_{ikmj} \ge Z_{ik} + Z_{jm} - 1. \tag{6}$$

Based on the aforementioned analyses, a mathematical model of the bi-objective fuzzy H&S network design problem can be formulated as (BFH&S, for short):

$$\begin{cases} \min & [\mathrm{C}(\mathbf{X}, \mathbf{Z}), \mathrm{T}(\mathbf{X}, \mathbf{Z})] \\ \text{s.t.:} & \tilde{\mathrm{C}}\mathrm{r}\left\{(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj})X_{ikmj} \le R\right\} \ge \alpha, \forall i, j, k, m \in N \\ & X_{ikmj} \ge Z_{ik} + Z_{jm} - 1, \forall i, j, k, m \in N \\ & \sum_{k\in N} Z_{ik} = 1, \forall i \in N \\ & Z_{ik} \le Z_{kk}, \forall i, k \in N \\ & \sum_{k\in N} Z_{kk} = p \\ & Z_{ik} \in \{0, 1\}, \forall i, k \in N \\ & X_{ikmj} \in \{0, 1\}, \forall i, j, k, m \in N. \end{cases} \tag{7}$$

This BFH&S problem is based on the classical hub center problem [4]. Several novelties made by this paper in comparison to Campbell [4] are emphasized here. First, in view of the studied problems, we study a BFH&S problem that attempts to address the multidimensional nature of the hub center problem proposed in Campbell [4] by considering the transportation cost and travel time simultaneously. Second, in terms of the proposed models, we adopt T2 fuzzy variables to capture the uncertainty associated with the transportation cost and travel time, whereas Campbell [4] proposed a deterministic model. Third, from the perspective of decision-making criteria, this paper uses the generalized expectation criterion and VaR criterion to characterize the performance of the IH&S network, which were not given by Campbell [4]. In summary, the proposed BFH&S problem is a significant and non-trivial extension because it can be used more flexibly to reflect the company's philosophy of modeling uncertainty.

4. Theoretical analyses of the proposed model

To solve Model (7), it is necessary to compute the generalized expectations of fuzzy variables in the objective and generalized credibility of fuzzy event in the constraints. Therefore, the solution methods for Model (7) require conversion of the generalized expected objective and credibility constraints to their respective parametric equivalents. However, this conversion is generally difficult to perform for general T2 fuzzy travel times and demands and is only feasible in some special cases. In this section, we discuss some special cases, where the transportation costs and travel times are characterized by asymmetric T2 normal fuzzy variables, and transform the objective and constraints to their equivalent parametric forms.

4.1. Equivalent parametric form of the generalized expected objective

In this section, we consider the case in which the transportation costs are characterized by asymmetric T2 normal fuzzy variables, and we derive the analytical expressions for the expected value for the fuzzy variables reduced by MV reduction methods. Applying the analytical expressions, we can transform the generalized expected objective into its equivalent parametric form.

Theorem 3. *Let the flow shipment cost* $\widetilde{C}_{ij} = \tilde{n}(\nu_{ij}, \rho_{ij}^2; \theta_1^l, \theta_1^r)$ *and the set-up cost* $\widetilde{F}_k = \tilde{n}(\nu_k, \rho_k^2; \theta_1^l, \theta_1^r)$ *be asymmetric T2 normal fuzzy variables. Suppose that the reduced fuzzy variables by* E_*, E *and* E^* *reduction methods are mutually independent. Then, we have the following:*

(i) *With the* E_* *reduction method, the generalized expected objective in Model (7) is equivalent to*

$$\widetilde{E}\left[\sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\widetilde{C}_{ik} + d_1\widetilde{C}_{km} + \widetilde{C}_{mj})X_{ikmj} + \sum_{k\in N}\widetilde{F}_k Z_{kk}\right]$$
$$= \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} \frac{(2-\theta_1^l)}{2} W_{ij}(\nu_{ik} + d_1\nu_{km} + \nu_{mj})X_{ikmj} + \sum_{k\in N}\frac{(2-\theta_1^l)}{2}\nu_k Z_{kk}.$$

(ii) *With the* E *reduction method, the generalized expected objective in Model (7) is equivalent to*

$$\widetilde{E}\left[\sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\widetilde{C}_{ik} + d_1\widetilde{C}_{km} + \widetilde{C}_{mj})X_{ikmj} + \sum_{k\in N}\widetilde{F}_k Z_{kk}\right]$$
$$= \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} \frac{(4-\theta_1^l)}{4} W_{ij}(\nu_{ik} + d_1\nu_{km} + \nu_{mj})X_{ikmj} + \sum_{k\in N}\frac{(4-\theta_1^l)}{4}\nu_k Z_{kk}.$$

(iii) *With the* E^* *reduction method, the generalized expected objective in Model (7) is equivalent to*

$$\widetilde{E}\left[\sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\widetilde{C}_{ik} + d_1\widetilde{C}_{km} + \widetilde{C}_{mj})X_{ikmj} + \sum_{k\in N}\widetilde{F}_k Z_{kk}\right]$$
$$= \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\nu_{ik} + d_1\nu_{km} + \nu_{mj})X_{ikmj} + \sum_{k\in N}\nu_k Z_{kk}.$$

Proof. We only prove case (ii), and the rest cases can be proved similarly.

Let C_{ij} and F_k be the reductions of T2 fuzzy variables $\widetilde{C}_{ij}$ and $\widetilde{F}_k$ by the E method. Then, their parametric possibility distributions are

$$\mu_{C_{ij}}(x;\theta_l,\theta_r) = \frac{(4-\theta_1^l-\theta_1^r)}{4}\exp\left(-\frac{(x-\nu_{ij})^2}{2\rho_{ij}^2}\right) + \frac{\theta_1^r}{4},$$

and

$$\mu_{F_k}(x;\theta_l,\theta_r) = \frac{(4-\theta_1^l-\theta_1^r)}{4}\exp\left(-\frac{(x-\nu_k)^2}{2\rho_k^2}\right) + \frac{\theta_1^r}{4}.$$

According to the definition of generalized credibility measure, we can calculate that the E reduced fuzzy variables C_{ij} and F_k have the following credibility distribution functions

$$\tilde{\mathrm{C}}\mathrm{r}\{C_{ij} \ge x\} = \begin{cases} \frac{4-\theta_1^l}{4} - \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_{ij})^2}{2\rho_{ij}^2}\right) - \frac{\theta_1^r}{8}, \text{if } x \le \nu_{ij} \\ \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_{ij})^2}{2\rho_{ij}^2}\right) + \frac{\theta_1^r}{8}, \text{if } x > \nu_{ij}, \end{cases}$$

$$\tilde{\mathrm{Cr}}\{C_{ij} \le x\} = \begin{cases} \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_{ij})^2}{2\rho_{ij}^2}\right) + \frac{\theta_1^r}{8}, \text{if } x \le \nu_{ij} \\ \frac{4-\theta_1^l}{4} - \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_{ij})^2}{2\rho_{ij}^2}\right) - \frac{\theta_1^r}{8}, \text{if } x > \nu_{ij}, \end{cases}$$

$$\tilde{\mathrm{Cr}}\{F_k \ge x\} = \begin{cases} \frac{4-\theta_1^l}{4} - \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_k)^2}{2\rho_k^2}\right) - \frac{\theta_1^r}{8}, \text{if } x \le \nu_k \\ \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_k)^2}{2\rho_k^2}\right) + \frac{\theta_1^r}{8}, \text{if } x > \nu_k, \end{cases}$$

and

$$\tilde{\mathrm{Cr}}\{F_k \le x\} = \begin{cases} \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_k)^2}{2\rho_k^2}\right) + \frac{\theta_1^r}{8}, \text{if } x \le \nu_k \\ \frac{4-\theta_1^l}{4} - \frac{(4-\theta_1^l-\theta_1^r)}{8}\exp\left(-\frac{(x-\nu_k)^2}{2\rho_k^2}\right) - \frac{\theta_1^r}{8}, \text{if } x > \nu_k. \end{cases}$$

By the definition of the generalized expected value, we have the following computational results

$$\tilde{E}[C_{ij}] = \int_0^{+\infty} \tilde{\mathrm{Cr}}\{C_{ij} \ge r\}\mathrm{d}x - \int_{-\infty}^0 \tilde{\mathrm{Cr}}\{C_{ij} \le r\}\mathrm{d}x = \frac{(4-\theta_1^l)\nu_{ij}}{4},$$

and

$$\tilde{E}[F_k] = \int_0^{+\infty} \tilde{\mathrm{Cr}}\{F_k \ge r\}\mathrm{d}x - \int_{-\infty}^0 \tilde{\mathrm{Cr}}\{F_k \le r\}\mathrm{d}x = \frac{(4-\theta_1^l)\nu_k}{4}.$$

Since C_{ij} and F_k are mutually independent, according to [18], we have

$$\begin{aligned} &\tilde{E}\left[\sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\tilde{C}_{ik} + d_1\tilde{C}_{km} + \tilde{C}_{mj})X_{ikmj} + \sum_{k\in N}\tilde{F}_k Z_{kk}\right] \\ &= \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}(\tilde{E}[C_{ik}] + d_1\tilde{E}[C_{km}] + \tilde{E}\big[C_{mj}\big])X_{ikmj} + \sum_{k\in N}\tilde{E}[F_k]Z_{kk} \\ &= \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} \frac{(4-\theta_1^l)}{4} W_{ij}(\nu_{ik} + d_1\nu_{km} + \nu_{mj})X_{ikmj} + \sum_{k\in N}\frac{(4-\theta_1^l)}{4}\nu_k Z_{kk}. \end{aligned}$$

which completes the proof of case (ii). □

4.2. Equivalent parametric form of the generalized credibility constraints

We now consider the case in which the travel times are characterized by asymmetric T2 normal fuzzy variables, and we transform the generalized credibility constraints into their equivalent parametric constraint forms.

Theorem 4. *Let the travel time $\tilde{T}_{ij} = \tilde{n}(\mu_{ij}, \sigma_{ij}^2; \theta_2^l, \theta_2^r)$ be an asymmetric T2 normal fuzzy variable. Suppose that the reduced fuzzy variables by E_*, E and E^* reduction methods are mutually independent. Then, we have the following:*

(i) *Using the E_* reduction method, if $0 < \alpha \le 1/2$, then $\tilde{\mathrm{Cr}}\{(\tilde{T}_{ik} + d_2\tilde{T}_{km} + \tilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ is equivalent to*

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} - (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{4\alpha}{2-\theta_2^l}}\right)X_{ikmj} \le R, \forall i, j, k, m \in N,$$

and if $1/2 < \alpha \le (2-\theta_l)/2$, then $\tilde{\mathrm{Cr}}\{(\tilde{T}_{ik} + d_2\tilde{T}_{km} + \tilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ is equivalent to

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} + (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{4-4\alpha-2\theta_2^l}{2-\theta_2^l}}\right)X_{ikmj} \le R, \forall i, j, k, m \in N.$$

(ii) *Using the E reduction method, if $\theta_2^r/4 < \alpha \le 1/2$, then $\tilde{\mathrm{Cr}}\{(\tilde{T}_{ik} + d_2\tilde{T}_{km} + \tilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ is equivalent to*

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} - (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8\alpha-\theta_2^r}{4-\theta_2^l-\theta_2^r}}\right)X_{ikmj} \le R, \forall i, j, k, m \in N,$$

and if $1/2 < \alpha \le (4-\theta_2^l)/4$, then $\tilde{\mathrm{Cr}}\{(\tilde{T}_{ik} + d_2\tilde{T}_{km} + \tilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ is equivalent to

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} + (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8-8\alpha-2\theta_2^l-\theta_2^r}{4-\theta_2^l-\theta_2^r}}\right)X_{ikmj} \le R, \forall i, j, k, m \in N.$$

(iii) *Using the* E^* *reduction method, if* $\theta_2^r/2 < \alpha \le 1/2$, *then* $\tilde{\mathrm{Cr}}\{(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ *is equivalent to*

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} - (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{4\alpha - \theta_2^r}{2 - \theta_2^r}}\right)X_{ikmj} \le R, \forall i, j, k, m \in N,$$

and if $1/2 < \alpha \le 1$, *then* $\tilde{\mathrm{Cr}}\{(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ *is equivalent to*

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} + (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{4 - 4\alpha}{2 - \theta_2^r}}\right)X_{ikmj} \le R, \forall i, j, k, m \in N.$$

Proof. We only prove case (ii), and the other two cases can be proved similarly.

Let T_{ij} be the reductions of T2 fuzzy variable $\widetilde{T}_{ij}$ by the E reduction method. Then, the parametric possibility distribution of the travel time T_{ij} is

$$\mu_{T_{ij}}(x; \theta_2^l, \theta_2^r) = \frac{(4 - \theta_2^l - \theta_2^r)}{4}\exp\left(-\frac{(x - \mu_{ij})^2}{2\sigma_{ij}^2}\right) + \frac{\theta_2^r}{4}.$$

Thus, the λ-cut set of T_{ij} is $T_{ij,\lambda} = [T_{ij,\lambda}^L, T_{ij,\lambda}^R]$, where

$$T_{ij,\lambda}^L = \mu_{ij} - \sigma_{ij}\sqrt{-2\ln\frac{4\lambda - \theta_2^r}{4 - \theta_2^l - \theta_2^r}},$$

and

$$T_{ij,\lambda}^R = \mu_{ij} + \sigma_{ij}\sqrt{-2\ln\frac{4\lambda - \theta_2^r}{4 - \theta_2^l - \theta_2^r}},$$

for $\theta_2^r/4 \le \lambda \le (4 - \theta_2^l)/4$.

For simplicity of presentation, we write $\xi_{ikmj} = T_{ik} + d_2T_{km} + T_{mj}$. By the supposition $0 \le \theta_2^l \le 1$ and $0 \le \theta_2^r \le 1$, we then have

$$0 \le \theta_2^r/4 \le 1/4, \quad 3/4 \le (4 - \theta_2^l)/4 \le 1.$$

Since the travel times T_{ik}, T_{km} and T_{mj} are mutually independent, by the properties of independence [18], we get the λ-cut of ξ_{ikmj} as follows

$$\xi_{ikmj,\lambda} = [\xi_{ikmj,\lambda}^L, \xi_{ikmj,\lambda}^R] = [T_{ik,\lambda}^L + dT_{km,\lambda}^L + T_{jm,\lambda}^L, T_{ik,\lambda}^R + dT_{km,\lambda}^R + T_{jm,\lambda}^R].$$

In the case of $\theta_2^r/4 < \alpha \le 1/2$, we have

$$\tilde{\mathrm{Cr}}\{\xi_{ikmj} \le R\} \ge \alpha \Longleftrightarrow R \ge \xi_{ikmj,2\alpha}^L,$$

where $\xi_{ikmj,2\alpha}^L$ is the right extreme point of the 2α-cut set of ξ_{ikmj}.

Hence, $\tilde{\mathrm{Cr}}\{\xi_{ikmj} \le R\} \ge \alpha$ is equivalent to

$$\mu_{ik} + d_2\mu_{km} + \mu_{mj} - (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8\alpha - \theta_2^r}{4 - \theta_2^l - \theta_2^r}} \le R.$$

It then follows that the generalized credibility constraint $\tilde{\mathrm{Cr}}\{(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ can be express as

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} - (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8\alpha - \theta_2^r}{4 - \theta_2^l - \theta_2^r}}\right)X_{ikmj} \le R.$$

Similarly, in the case of $1/2 < \alpha \le (4 - \theta_2^l)/4$, one has

$$\tilde{\mathrm{Cr}}\{\xi_{ikmj} \le R\} \ge \alpha \Longleftrightarrow R \ge \xi_{ikmj,(4-\theta_2^l)/2-2\alpha}^R,$$

where $\xi_{ikmj,(4-\theta_2^l)/2-2\alpha}^R$ is the right extreme point of the $((4 - \theta_2^l)/2 - 2\alpha)$-cut set of ξ_{ikmj}.

As a consequence, $\tilde{\mathrm{C}}\mathrm{r}\{\xi_{ikmj} \le R\} \ge \alpha$ is equivalent to

$$\mu_{ik} + d_2\mu_{km} + \mu_{mj} + (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8-8\alpha-2\theta_2^l-\theta_2^r}{4-\theta_2^l-\theta_2^r}} \le R.$$

It then follows that the generalized credibility constraint $\tilde{\mathrm{C}}\mathrm{r}\{(\widetilde{T}_{ik} + d_2\widetilde{T}_{km} + \widetilde{T}_{mj})X_{ikmj} \le R\} \ge \alpha$ can be written as

$$\left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} + (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8-8\alpha-2\theta_2^l-\theta_2^r}{4-\theta_2^l-\theta_2^r}}\right)X_{ikmj} \le R.$$

The proof of the assertion (ii) is complete. □

5. Solution method

To solve this model, a two-phase approach is developed in this section. The first phase converts the proposed model (7) into its equivalent parametric mixed-integer programming problems by applying an equivalent transformation method. In the second phase, a fuzzy linear programming approach implemented with an augmented max-min operator is utilized to obtain a non-dominated solution.

5.1. Phase 1: Equivalent transformation method

In this section, we will take the E reduction method as an example to illustrate the equivalent transformation method for deriving the equivalent parametric programming problems.

According to Theorem 4, the equivalent parametric representations of generalized credibility constraints are available in two cases, and each situation is determined by the parametric domain of variable α. Consequently, the original optimization model (7) can be decomposed into two equivalent parametric programming sub-models, described as follows.

In the case of $\theta_2^r/4 < \alpha \le 1/2$, we can transform Model (7) into the following equivalent parametric programming problem according to Theorems 3 and 4:

$$\begin{cases} \min & [\mathrm{C}(\mathbf{X},\mathbf{Z}), \mathrm{T}(\mathbf{X},\mathbf{Z})] \\ \text{s.t.:} & \left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} - (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8\alpha-\theta_2^r}{4-\theta_2^l-\theta_2^r}}\right)X_{ikmj} \le R, \forall i,j,k,m \in N \\ & \text{Constraints}(3)-(6), \end{cases} \tag{8}$$

where $\mathrm{C}(\mathbf{X},\mathbf{Z}) = \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N}\frac{(4-\theta_1^l)}{4}W_{ij}(\nu_{ik} + d_1\nu_{km} + \nu_{mj})X_{ikmj} + \sum_{k\in N}\frac{(4-\theta_1^l)}{4}\nu_k Z_{kk}$ and $\mathrm{T}(\mathbf{X},\mathbf{Z}) = R$.

In the case of $1/2 < \alpha \le (4-\theta_2^l)/4$, we can transform Model (7) into the following equivalent parametric programming problem according to Theorems 3 and 4:

$$\begin{cases} \min & [\mathrm{C}(\mathbf{X},\mathbf{Z}), \mathrm{T}(\mathbf{X},\mathbf{Z})] \\ \text{s.t.:} & \left(\mu_{ik} + d_2\mu_{km} + \mu_{mj} + (\sigma_{ik} + d_2\sigma_{km} + \sigma_{mj})\sqrt{-2\ln\frac{8-8\alpha-2\theta_2^l-\theta_2^r}{4-\theta_2^l-\theta_2^r}}\right)X_{ikmj} \le R, \forall i,j,k,m \in N \\ & \text{Constraints}(3)-(6), \end{cases} \tag{9}$$

where $\mathrm{C}(\mathbf{X},\mathbf{Z}) = \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N}\frac{(4-\theta_1^l)}{4}W_{ij}(\nu_{ik} + d_1\nu_{km} + \nu_{mj})X_{ikmj} + \sum_{k\in N}\frac{(4-\theta_1^l)}{4}\nu_k Z_{kk}$ and $\mathrm{T}(\mathbf{X},\mathbf{Z}) = R$.

Since the parametric domain of variable α is divided into two subregions according to the values of parameters, the solution process is performed by solving two different sub-models, followed by comparing the optimal solutions to the two sub-models. Accordingly, we refer to this solution process as the equivalent transformation method.

5.2. Phase 2: Fuzzy linear programming approach

Here, we make use of the result in phase 1 to solve multi-objective optimization problems by the fuzzy linear programming approach, which models each objective as a fuzzy set whose membership function represents the satisfaction degree of the objective. The membership degree is generally assumed to increase linearly from zero (for the smallest satisfactory value) to one (for the highest satisfactory value). Zimmermann [42] first used the max-min operator to aggregate the fuzzy objectives for making a compromise decision. However, it cannot guarantee a non-dominated solution and is not completely compensatory. To achieve full compensation between aggregated membership functions and to obtain a non-dominated solution, we use the extended max-min approach [14,15].

First, it is easy to calculate the range for each objective by the single-objective optimization methods. Here, we use $\mathrm{C}_{\min}$ and $\mathrm{C}_{\max}$ to denote the minimum and maximum total transportation costs, respectively, and we use $\mathrm{T}_{\min}$ and $\mathrm{T}_{\max}$

to denote the minimum and maximum travel time requirements, respectively. Furthermore, we construct the membership functions for the financial objective and service objective as follows:

$$\mu_C(x) = \begin{cases} 1, & \text{if } x < C_{\min} \\ \frac{C_{\max}-x}{C_{\max}-C_{\min}}, & \text{if } C_{\min} \le x \le C_{\max} \\ 0, & \text{if } x > C_{\max}, \end{cases}$$

and

$$\mu_T(x) = \begin{cases} 1, & \text{if } x < T_{\min} \\ \frac{T_{\max}-x}{T_{\max}-T_{\min}}, & \text{if } T_{\min} \le x \le T_{\max} \\ 0, & \text{if } x > T_{\max}. \end{cases}$$

Finally, we aggregate $\mu_C(x)$ and $\mu_T(x)$ by using the augmented max-min operator and then formulate the following single-objective optimization model:

$$\begin{cases} \max & \lambda + \epsilon(\mu_C(x) + \mu_T(x))/2 \\ \text{s.t.:} & \mu_C(x) \ge \lambda \\ & \mu_T(x) \ge \lambda \\ & \text{Constraints } (3)-(6), \end{cases} \tag{10}$$

where λ is an auxiliary variable that represents the overall satisfactory level of compromise (to be maximized) and ϵ is a small positive number. Note that a non-dominated solution can be always generated when λ is maximized. The single-objective model (10) can be effectively solved by a Branch and Bound solver of a general-purpose optimization software, LINGO [2].

5.3. Algorithm: Two-phase approach

The following solution procedure is employed to solve the proposed fuzzy bi-objective H&S network design problem. In this way, the steps of the proposed solving approach can be summarized as follows.

Phase 1

Step 1: Identify the parameters θ_1^l, θ_1^r, θ_2^l, θ_2^r and α.
Step 2: Convert Model (7) into the equivalent parametric multi-objective programming model using the equivalent transformation method.

Phase 2

Step 3: Calculate the range for each objective ($[C_{\min}, C_{\max}]$ and $[T_{\min}, T_{\max}]$) with an optimization technique such as the Branch and Bound method embedded in LINGO software.
Step 4: Define the membership function of each goal in the fuzzy bi-objective linear programming.
Step 5: Transform the equivalent parametric programming multi-objective model into a single-objective model by using the extended max-min operator.
Step 6: Specify the value of ϵ (0.05) and solve the respective single-objective model using LINGO software to obtain an optimal solution.

From the above algorithm, it should be evident that the approach is effective in obtaining a non-dominated solution that has equal satisfactory degree on both objectives.

Remark 2. When the T2 fuzzy transportation costs and travel times are reduced by the E^* reduction method or E_* reduction method, we can obtain the optimal solutions to Model (7) using solution procedures similar to those described above.

6. Numerical experiments

To show the performance of the proposed approach, a set of numerical experiments will be implemented in this section. The involved problems are solved by a Branch and Bound solver of LINGO software, on a personal computer (Lenovo with Intel(R) Core(TM) i3-4170 CPU @ 3.70 GHz and 4.00 GB RAM), using the Microsoft Windows 7 operating system.

6.1. Case study representation

We apply our approach to a case study of the H&S network design on the Civil Aeronautics Board (CAB) data set introduced by O'Kelly [28], which contains O-D flows and air transportation costs for 25 cities in the US air network (see Fig. 3) and can be downloaded from the OR-Library [29]. This data set is modified to generate data for solving the proposed BFH&S problem under T2 fuzzy uncertainty by using the following changes. In detail, we adopt asymmetric T2 normal fuzzy variables to represent the uncertain flow shipment costs, set-up costs and travel times. For each node pair $i, j \in N$, we assume

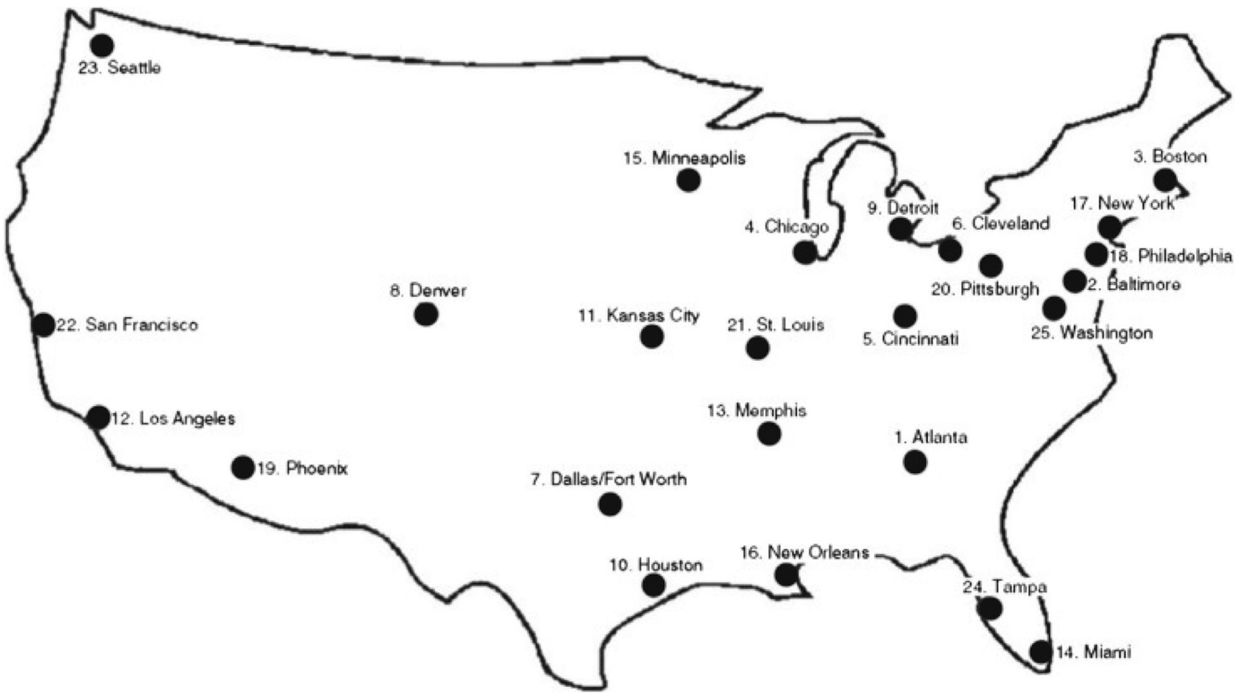

Fig. 3. US cities in the CAB data set.

that uncertain flow shipment costs and set-up costs are characterized by $\widetilde{C}_{ij} = \tilde{n}(\bar{c}_{ij}, \rho_1^2; \theta_1^l, \theta_1^r)$ and $\widetilde{F}_k = \tilde{n}(\bar{c}_k, \rho_2^2; \theta_1^l, \theta_1^r)$, respectively, where $\bar{c}_{ij}$ and $\bar{c}_k$ are the corresponding deterministic shipment costs and set-up costs given in the CAB data set and ρ_1 and ρ_2 are randomly generated in the interval [0, 1] according to a uniform distribution. This range is used for computational purposes to reflect the uncertainty in flow shipment costs and set-up costs. The uncertain travel time between cities i and j is described as $\widetilde{T}_{ij} = \tilde{n}(\bar{t}_{ij}, 1; \theta_2^l, \theta_2^r)$, where $\bar{t}_{ij}$ is sampled from a $U(10, 100)$ distribution for travel time data (not provided in the CAB data set). The reasoning behind the choice of this interval is associated with the need to obtain data with some relevant degree of travel time variability. Concerning the demand for each node pair $i, j \in N$, we set $W_{ij} = \overline{w}_{ij}$, where $\overline{w}_{ij}$ is the deterministic demand given in the CAB data set. Moreover, we set $\theta_l = \theta_1^l = \theta_2^l$ and $\theta_r = \theta_1^r = \theta_2^r$, where θ_l and θ_r are two parameters characterizing the degree of fuzzy uncertainty. The parameters θ_l and θ_r are randomly chosen from the set [0, 1].

For the generated data set, we consider several experiments by varying the number of hubs $p \in \{2, 3, 4\}$. For each hub number value, we provide results with two levels of inter-hub discount $d = d_1 = d_2 = 0.2$ and 0.8. Each instance has been solved for three different generalized credibility levels $\alpha \in \{0.4, 0.6, 0.8\}$.

6.2. Computational results

Tables 1–3 summarize the computational results by the E_*, E and E^* reduction methods for this case study, respectively. For each value of discount factor (d) in these tables, the columns labeled "Obj-Cost" and "Obj-Time" provide the optimal value of the objective functions for each tested cases. The next column, called "Open hubs", displays the locations at which the hub facilities are established in the optimal solution. The results from the above experiments provide us with valuable information on how, for the same instance, the configuration of the H&S network associated with the optimal solution changes depending on values of the parameters θ_l and θ_r. It can be observed that the objective functions increase as the generalized credibility level increases. This is an expected result because as the generalized credibility level increases, the company would spend relatively more transportation cost and set a longer travel time requirement to meet customers satisfaction. It is also observed from Tables 1–3 that when the value of d becomes greater, the values of both objective functions increase. It is intuitive that as the level of inter-hub discount becomes higher, the company would pay higher transportation costs and take more travel time for the flow traversing between the hub nodes. Therefore, with the BFH&S model proposed in this paper, the company can obtain more precise information and make better decisions.

Figs. 4 and 5 illustrate the optimal network configurations by the E reduction method on the CAB data set using $d = 0.2$, $\alpha = 0.8$, $\theta_l = 0.5$ and $\theta_l = 0.5$. Fig. 4 illustrates the hub topology with two hubs located in Cincinnati and Denver and corresponding allocated cities. Denver appears to be the hub for the central and eastern parts of the United States, whereas Cincinnati is responsible for the western parts of the country. Furthermore, Fig. 5 shows the hub topology with 3 hubs in

Table 1
Computational results by E_* reduction method.

p	θ_l	α	$d = 0.2$			$d = 0.8$		
			Open hubs	Obj-Cost	Obj-Time	Open hubs	Obj-Cost	Obj-Time
2	0.2	0.4	5,8	902.34	114.19	5,8	1213.17	145.21
		0.6	5,8	1052.46	130.22	5,8	1372.63	162.46
		0.8	8,20	1211.33	146.58	8,20	1509.42	179.23
	0.3	0.4	8,25	797.29	92.36	8,25	1081.25	125.75
		0.6	8,25	935.11	105.44	8,25	1229.43	142.34
		0.8	5,8	1100.77	117.35	5,8	1352.97	156.48
3	0.5	0.4	1,4,8	1425.56	103.65	1,4,8	1705.39	131.75
		0.6	1,4,8	1603.19	115.73	1,4,8	1879.28	140.39
		0.8	1,5,8	1799.34	129.57	1,5,8	1934.77	153.78
	0.7	0.4	1,8,20	1216.55	81 43	1,8,20	1517.34	110.67
		0.6	1,8,20	1355.11	94.36	1,8,20	1622.21	122.93
		0.8	1,5,8	1572.89	103.26	1,5,8	1754.55	135.47
4	0.8	0.4	1,4,8,20	1849.13	90.48	1,4,8,20	2210.57	119.87
		0.6	1,4,8,20	2017.19	101.67	1,4,8,20	2453.18	125.45
		0.8	1,4,5,8	2199.43	112.95	1,4,5,8	2588.19	139.71
	1.0	0.4	1,5,8,20	1656.73	71.56	1,5,8,20	2001.43	93.33
		0.6	1,5,8,20	1795.34	84.43	1,5,8,20	2172.87	104.92
		0.8	1,5,8,25	1945.38	93.28	1,5,8,25	2294.13	117.28

Table 2
Computational results by E reduction method.

p	(θ_l, θ_r)	α	$d = 0.2$			$d = 0.8$		
			Open hubs	Obj-Cost	Obj-Time	Open hubs	Obj-Cost	Obj-Time
2	(0.2,0.8)	0.4	8,20	1142.19	125.46	8,20	1407.46	156.36
		0.6	8,20	1260.23	141.67	8,20	1565.39	174.38
		0.8	5,8	1403.71	157.34	5,8	1698.16	190.45
	(0.3,0.7)	0.4	5,8	984.35	102.11	5,8	1275.56	136.49
		0.6	5,8	1128.45	114.98	5,8	1415.38	155.42
		0.8	8,25	1285.17	128.45	8,25	1532.67	164.11
3	(0.5,0.5)	0.4	1,5,8	1651.88	113.75	1,5,8	1900.39	142.19
		0.6	1,5,8	1827.43	126.43	1,5,8	2054.11	153.05
		0.8	1,8,20	1992.52	140.99	1,8,20	2176.45	164.11
	(0.7,0.3)	0.4	1,4,8	1465.59	92.77	1,4,8	1734.98	121.55
		0.6	1,4,8	1547.31	105.42	1,4,8	1839.22	135.66
		0.8	1,5,8	1702.46	112.65	1,5,8	1963.35	147.13
4	(0.8,0.2)	0.4	1,4,5,8	2001.46	101.76	1,4,5,8	2405.79	130.06
		0.6	1,4,5,8	2217.37	113.55	1,4,5,8	2621.55	139.45
		0.8	1,5,8,20	2397.45	124.07	1,5,8,20	2745.22	151.37
	(1.0,1.0)	0.4	1,4,8,20	1842.37	83.22	1,4,8,20	2207.13	104.65
		0.6	1,4,8,20	1967.33	96.32	1,4,8,20	2365.93	116.33
		0.8	1,4,8,25	2142.76	102.94	1,4,8,25	2501.13	129.06

Atlanta, Pittsburgh and Denver, which are selected to cover the entire country. It is observed that the location of hubs has been dispersed equitably through different cities to respect the travel time requirement for the customers.

6.3. Comparison of solutions

To further validate the advantages of the proposed model under the T2 fuzzy environment, more computational results were obtained to solve a similar BFH&S problem under a T1 fuzzy environment. In the case of $\theta_l = 0$ and $\theta_r = 0$, the T2 fuzzy shipment costs $\widetilde{C}_{ij}$, set-up costs $\widetilde{F}_k$ and travel times $\widetilde{T}_{ij}$ are degenerated to T1 fuzzy shipment costs c_{ij}, set-up costs c_k and travel times t_{ij} with fixed possibility distributions, respectively. The two-phase method described in Section 5 is also used to solve the T1 fuzzy model. Tables 4 and 5 summarize the comparison of solution results by E reduction method on the CAB data set using $\theta_l = 0.5$ and $\theta_l = 0.5$. It can be observed that the total transportation costs and the maximum travel time requirement under a T1 fuzzy environment are higher than those under the T2 fuzzy environment in terms of optimal solutions. This result suggests that the BFH&S problem under a T2 fuzzy environment obtains better results compared to its counterpart in a T1 fuzzy environment.

Next, we compare the results provided by the T2 fuzzy model with the results provided by the T1 fuzzy model about the discount factor and the generalized credibility level. For this purpose, we summarize the optimal hub topology in Figs. 6 and 7.

Table 3
Computational results by E^* reduction method.

p	θ_r	α	$d = 0.2$			$d = 0.8$		
			Open hubs	Obj-Cost	Obj-Time	Open hubs	Obj-Cost	Obj-Time
2	0.2	0.4	5,8	1339.17	136.34	5,8	1635.34	165.47
		0.6	5,8	1456.25	150.73	5,8	1773.21	182.33
		0.8	8,25	1600.13	168.22	8,25	1899.31	201.73
	0.3	0.4	4,8	1172.99	112.49	4,8	1473.05	146.55
		0.6	4,8	1301.45	123.65	4,8	1609.59	163.98
		0.8	5,8	1473.28	137.28	5,8	1729.15	172.03
3	0.5	0.4	1,4,8	1845.46	124.36	1,4,8	2105.47	151.67
		0.6	1,4,8	2012.19	135.08	1,4,8	2269.11	163.18
		0.8	1,5,8	2173.28	151.49	1,5,8	2380.65	174.48
	0.7	0.4	1,8,20	1653.37	101.27	1,8,20	1944.58	132.09
		0.6	1,8,20	1738.24	114.93	1,8,20	2041.76	144.22
		0.8	1,5,8	1901.11	123.41	1,5,8	2157.03	156.03
4	0.8	0.4	1,4,8,20	2200.79	112.03	1,4,8,20	2613.78	141.71
		0.6	1,4,8,20	2403.18	122.99	1,4,8,20	2807.19	150.33
		0.8	1,4,8,25	2588.17	134.76	1,4,8,25	2937.03	161.25
	1.0	0.4	1,4,5,8	2037.27	93.86	1,4,5,8	2417.35	114.38
		0.6	1,4,5,8	2158.76	105.79	1,4,5,8	2558.11	125.07
		0.8	1,4,8,20	2336.44	112.09	1,4,8,20	2716.38	138.66

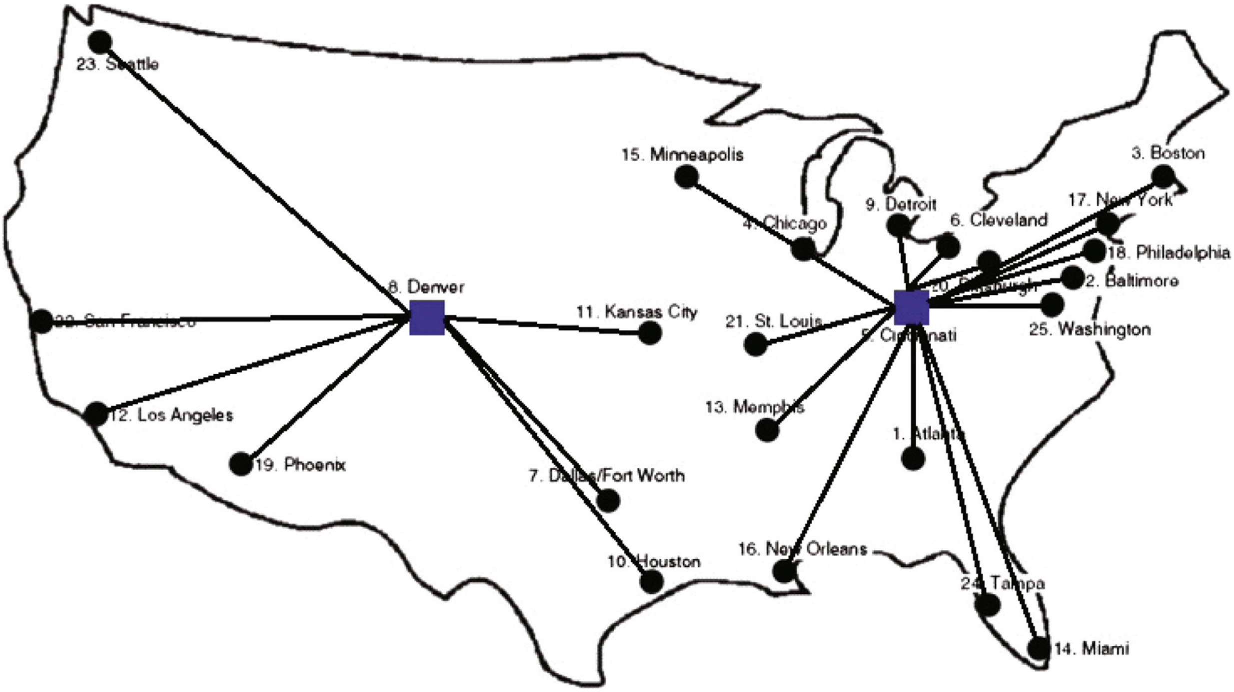

Fig. 4. H&S network topography for $p = 2$.

Table 4
Comparison of solution results with $d = 0.2$.

p	α	T1 fuzzy model			T2 fuzzy model		
		Open hubs	Obj-Cost	Obj-Time	Open hubs	Obj-Cost	Obj-Time
2	0.4	8,20	1731.92	175.33	5,8	**1239.44**	**124.52**
	0.6	8,20	1982.17	186.76	5,8	**1428.43**	**136.65**
	0.8	5,8	2197.38	197.43	8,20	**1589.37**	**151.11**
3	0.4	4,8,13	2413.28	164.44	1,5,8	**1651.88**	**113.75**
	0.6	4,8,13	2627.43	175.43	1,5,8	**1827.43**	**126.43**
	0.8	1,5,8	2829.32	186.33	1,8,20	**1992.52**	**140.99**
4	0.4	1,4,5,8	3019.38	152.43	1,4,8,20	**2249.44**	**104.21**
	0.6	1,4,5,8	3211.92	161.55	1,4,8,20	**2432.33**	**112.44**
	0.8	1,4,8,20	3409.63	170.56	1,4,8,25	**2587.36**	**120.39**

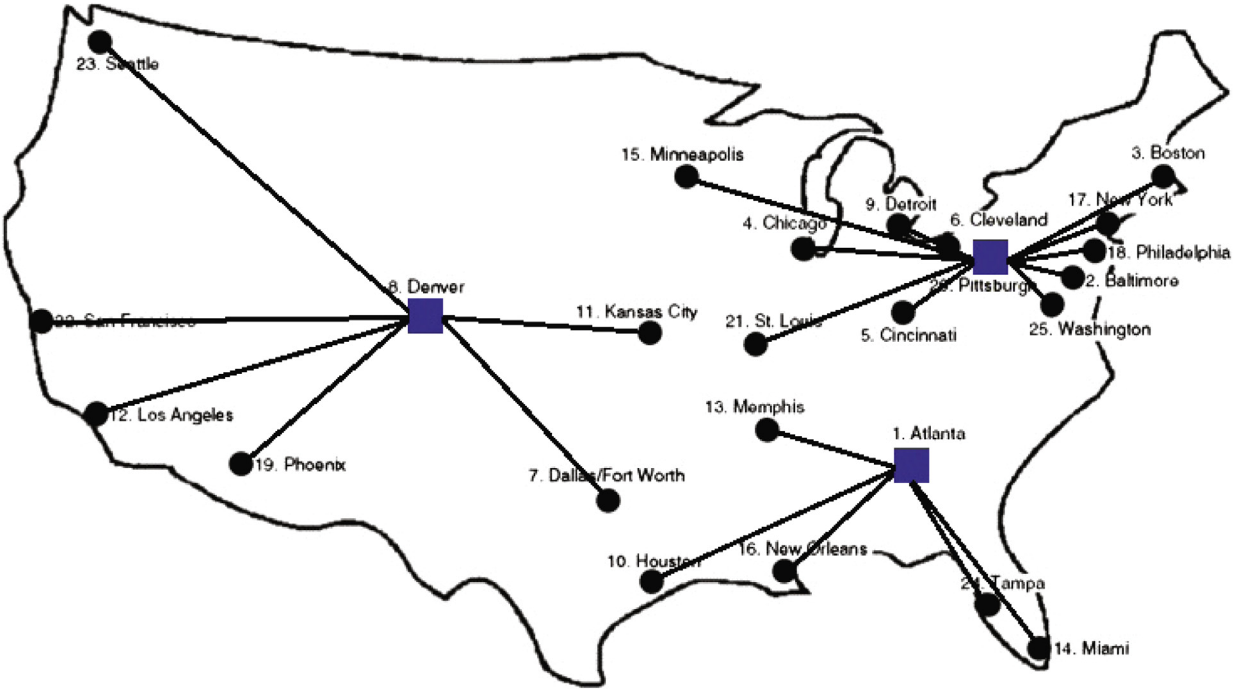

Fig. 5. H&S network topography for $p = 3$.

Table 5
Comparison of solution results with $d = 0.8$.

p	α	T1 fuzzy model			T2 fuzzy model		
		Open hubs	Obj-Cost	Obj-Time	Open hubs	Obj-Cost	Obj-Time
2	0.4	5,8	2022.56	202.49	5,8	**1542.97**	**153.33**
	0.6	5,8	2203.34	212.54	5,8	**1742.55**	**162.75**
	0.8	8,20	2367.76	221.35	8,20	**1899.37**	**171.46**
3	0.4	1,5,8	2705.65	184.77	1,5,8	**1900.39**	**142.19**
	0.6	1,5,8	2917.75	201.33	1,5,8	**2051.11**	**153.05**
	0.8	1,4,8	3121.66	216.67	1,8,20	**2176.45**	**164.11**
4	0.4	1,4,8,20	3321.54	180.21	1,4,8,20	**2513.65**	**131.66**
	0.6	1,4,8,20	3500.21	189.31	1,4,8,20	**2745.76**	**141.33**
	0.8	1,4,5,8	3689.11	197.53	1,4,8,25	**2847.21**	**150.21**

Fig. 6 plots the optimal solutions obtained by the T1 fuzzy model and T2 fuzzy model with different discount factors d and fixed generalized credibility levels $\alpha = 0.4$. From Fig. 6, we find that the network structure changes in the T1 fuzzy model. Specifically, when $d = 0.2$, the optimal hub cities are Pittsburgh and Denver. When $d = 0.8$, the optimal hub city Pittsburgh is changed to Cincinnati. However, Fig. 6 shows that there is no change in the results obtained by the T2 fuzzy model. Consequently, the hub locations in the T2 fuzzy model are robust with respect to the discount factor.

Fig. 7 depicts the optimal solutions obtained by the T1 fuzzy fuzzy model and T2 fuzzy model with different generalized credibility levels α and fixed discount $d = 0.2$. In Fig. 7, we observe that the network structure changes greatly in the T1 fuzzy model with different values of parameter α. For example, when $\alpha = 0.4$, the optimal hub cities are Chicago, Denver and Memphis. When $\alpha = 0.8$, the two optimal hub cities Chicago and Memphis are changed to Cincinnati and Atlanta. However, Fig. 7 shows that the optimal hub topology changes only one hub city in the T2 fuzzy model. Therefore, the hub locations in the T2 fuzzy model are robust with respect to the generalized credibility level.

From the comparison of solution results, we confirm that ignoring the inherent T2 fuzzy uncertainty involved in the model's parameters may lead to sub-optimality, whereas taking into account the possible changes in the data could guarantee finding a more reliable solution. Therefore, it can be concluded that the BFH&S problem under the T2 fuzzy environment is not only closer to reality than the T1 fuzzy environment but also can provide a more robust representation of the real H&S network.

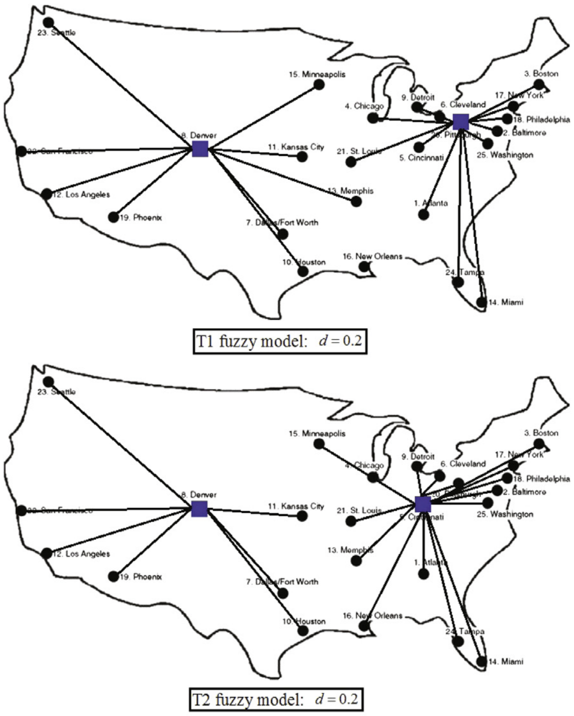

Fig. 6. Robustness analysis about the discount factor d.

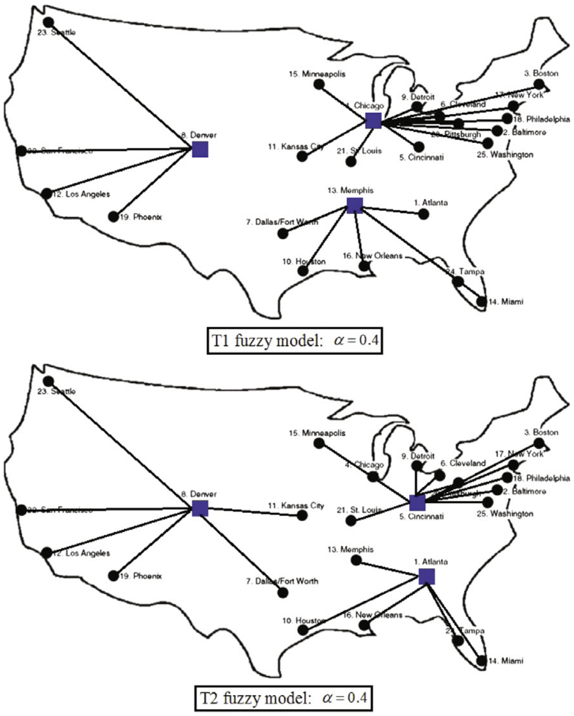

Fig. 7. Robustness analysis about the generalized credibility level α.

7. Conclusions

In this study, we addressed a bi-objective H&S network design problem, where the transportation costs and travel times were uncertain and characterized by parametric secondary possibility distributions. Two crucial objectives of the expected total transportation costs by using the generalized expectation and the maximum travel time requirement in terms of generalized VaR were considered for minimization. We proposed a two-phase approach for the fuzzy bi-objective H&S network design problem. In the first phase, we converted the proposed model into its equivalent parametric mixed-integer programming problems by applying an equivalent transformation method. We used a fuzzy linear programming approach implemented with an augmented max-min operator to obtain a non-dominated solution in the second phase. To show the effectiveness of the proposed approach, a case study was conducted. The numerical results demonstrated the superiority of the proposed model and optimization method.

This research can be extended by investigating additional issues, such as the effect of limited hub capacity and the impact of congestion on the design of H&S networks. Another direction for this research is to provide further understanding about the design of intermodal H&S networks. Our future research work will also focus on developing a dynamic programming algorithm to consider the real-time characteristics of data.

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