Physics Letters A 380 (2016) 3029–3035

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Pedestrian evacuation in view and hearing limited condition: The impact of communication and memory

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ARTICLE INFO

Article history:
Received 12 April 2016
Received in revised form 12 July 2016
Accepted 12 July 2016
Available online 16 July 2016
Communicated by C.R. Doering

Keywords:
Pedestrian evacuation
Communication
Perceptual field
Social force model

ABSTRACT

This paper studies pedestrian evacuation in view and hearing limited condition based on the social force approach. It is assumed that there are two types of pedestrians: Informed individuals know the exit location whereas uninformed individuals do not. The uninformed individuals can communicate with the informed ones within their perceptual fields, thus learning to know and memorize the exit location. We consider cases with and without communication/memory. The simulations show communication and memory are able to enhance the evacuation efficiency. We also investigate the impact of communication on the efficiency of an emergency exit.

1. Introduction

Pedestrian behavior as well as crowd dynamics has been modeled by various approaches from both the macroscopic and the microscopic level. The macroscopic models are concerned with the average quantities, in which pedestrians are not considered individually [1,2]. They are often adopted to assess the efficiency and safety of large-scale pedestrian facilities due to its high computational efficiency [3]. Compared with the macroscopic models, the microscopic models are more popular due to its good performance in reproducing individuals' behaviors as well as various self-organization phenomena. The social force model is one of the most widely adopted microscopic models, in which pedestrian's movement is governed by quantitative physical and social forces [4–6]. Many interesting features of the pedestrian crowd are reproduced with this model, such as the lane formation in bidirectional flows [4], oscillations at narrow bottlenecks [4], arching effect around the exit [5] and faster-is-slower effect [5]. The cellular automaton (CA) models describe pedestrian's movement in a gridded space at discrete time steps [7]. A successful CA based model for pedestrian evacuation is the floor field model [8,9], which consists of a static floor field to specify the attractive regions of the room for pedestrians and a dynamic floor field to imitate the virtual traces left by the leaving pedestrians. Some typical features of the pedestrian evacuation can also be reproduced in this model. Other microscopic models such as the velocity-based model [10], game-theoretic model [11], and a recent methodological approach based on operational methods of quantum mechanics [12] all contribute to increasing our understanding on pedestrian's behavior as well as crowd dynamics. For a detailed review of the existing modeling approaches for pedestrian dynamics, the readers may refer to Refs. [13,14].

Most of these existing models are dealing with the evacuation problem in a known environment, which means that most of the pedestrians have the complete information about the evacuation scenario. However, evacuation after earthquake, fire, or electricity blackouts is often accompanied by darkness, smoke, and scream. Under such circumstance, the pedestrians will have limited view and hearing range [15]. To study such situation, several simulation works have been performed [16–18], which shows that short view radius is adverse to evacuation. Moreover, experimental study on evacuation process from a room without visibility has also been reported [16], which shows that pedestrians prefer to move along the wall to find their way out.

In this paper, we study pedestrian evacuation in view and hearing limited condition. Under such circumstance, each individual has a limited perceptual field. Pedestrian's movement is governed by the social force model. It is assumed that there are two types of pedestrians: Informed individuals know the exit location whereas uninformed individuals do not. The uninformed individuals are able to communicate with the informed ones within their perceptual fields, thus learn to know and memorize the exit location. We also include a wall-following mechanism and herding behavior in the wayfinding process of the uninformed individuals. Therefore, for an informed individual, his/her desired moving direction is de-

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http://dx.doi.org/10.1016/j.physleta.2016.07.030
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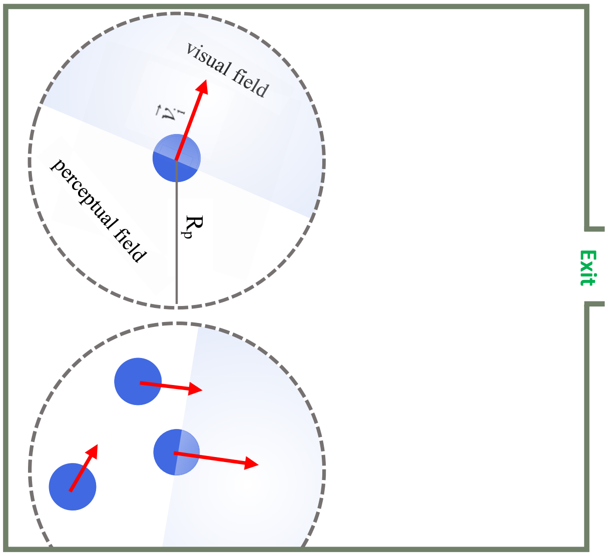

Fig. 1. Perceptual field and visual field of pedestrian.

termined by his/her memory information about the exit location; while for an uninformed individual, his/her desired moving direction is affected by the interactions with the informed neighbors via communication, wall-following behavior as well as herding behavior. We consider cases with and without communication/memory in a single-exit room, and the simulations show that communication and memory effect is able to enhance the evacuation efficiency. Finally, we investigate the impact of communication on the efficiency of an emergency exit.

We would like to mention that several papers have investigated effect of trained leaders on the evacuation process. It is assumed that trained leaders know the location of exit whereas others do not know [19–23]. Trained leaders will guide others to evacuate via communication.

Comparing with these studies, our study is novel at least in two aspects:

(1) The trained leaders are assumed to move with constant speed and direction. The main focus of the studies is on how fast the trained leaders should move, where should they stand initially, and how many trained leaders should there be [19–23]. As a result, the studies apply to evacuation in well prepared events. In contrast, our model applies to unexpected evacuation.
(2) In our model, the wall-following behavior, the herding behavior, communication and the memory effect have been taken into account, which have not been entirely considered in the studies mentioned above.

The paper is organized as follows. Section 2 presents the simulation model. The simulation results are presented and discussed in Section 3. The conclusion is given in Section 4.

2. Model

2.1. Perceptual field

This paper studies pedestrian evacuation process in view and hearing limited condition. Under such circumstance, each individual has a limited perceptual field. As shown in Fig. 1, we define that each pedestrian has a hearing field, which is a circular area with radius R_h, and a visual field, which is a semicircular area with radius R_v. The perceptual field is the combination of them, and for simplicity we assume $R_h = R_v = R_p$.

2.2. Social force model

We simulate pedestrian's movement by using the social force model. In this model, for a pedestrian i, his/her motion is determined by the superposition of his/her own desired force $\vec{f}_i^0$, the interaction force $\vec{f}_{ij}$ with other pedestrians j ($j \neq i$), and the interaction force $\vec{f}_{iW}$ with the wall W. For each pedestrian i of mass m_i at position $\vec{r}_i$, his/her motion is governed by the following equations [5]:

$$m_i \frac{d\vec{v}_i}{dt} = \vec{f}_i^0 + \sum_{j(\neq i)} \vec{f}_{ij} + \sum_W \vec{f}_{iW} \tag{1}$$

$$\vec{f}_i^0 = m_i \frac{v_i^0 \vec{e}_i^0 - v_i(t)}{\tau_i} \tag{2}$$

where the desired force $\vec{f}_i^0$ is used to describe pedestrian's incentive to move in a certain direction $\vec{e}_i^0$ with the desired speed v_i^0. Here $\vec{e}_i^0 = Norm(\vec{e}_i(t))$ with $Norm(\vec{u}) = \vec{u}/\|\vec{u}\|$ and $\vec{e}_i(t)$ is the desired escape direction that will be addressed in section 2.3. τ_i is the acceleration time, within which a pedestrian tends to approach his/her desired velocity $v_i^0 \vec{e}_i^0$.

The interaction force with other pedestrians is specified by three components: the repulsive force $\vec{f}_{ij}^r$ to describe the tendency to keep a situation-dependent distance to others; the body force $\vec{f}_{ij}^b$ due to body compression and the sliding friction force $\vec{f}_{ij}^s$ due to their relative motion when they are in touch with each other ($d_{ij} < r_{ij}$) [5]:

$$\vec{f}_{ij} = \vec{f}_{ij}^r + \vec{f}_{ij}^b + \vec{f}_{ij}^s \tag{3}$$

$$\vec{f}_{ij}^r = A_i \exp\big[(r_{ij} - d_{ij})/B_i\big]\vec{n}_{ij} z(\cos\varphi_{ij}) z(R_p - d_{ij}) \tag{4}$$

$$\vec{f}_{ij}^b = K g(r_{ij} - d_{ij})\vec{n}_{ij} \tag{5}$$

$$\vec{f}_{ij}^s = \kappa g(r_{ij} - d_{ij})\Delta v_{ji}^\tau \vec{t}_{ij} \tag{6}$$

where A_i, B_i are two constants, $r_{ij} = r_i + r_j$ is the sum of their radii r_i and r_j, and $n_{ij} = (n_{ij}^1, n_{ij}^2) = (\vec{r}_i - \vec{r}_j)/d_{ij}$ is the normalized vector pointing from pedestrian j to i with $d_{ij} = \|\vec{r}_i - \vec{r}_j\|$ being the distance between pedestrian i and j. The function $z(x)$ is equal to 1 if $x \geq 0$ and otherwise is zero, φ_{ij} is the angle between the direction of $\vec{v}_i$ and the direction $-\vec{n}_{ij}$, R_p is the radius of the perceptual field, and the term $z(\cos\varphi_{ij})z(R_p - d_{ij})$ is used to guarantee that pedestrian j is in the visual field of pedestrian i. In formula (5) and formula (6), K, κ are constants, $\vec{t}_{ij} = (-n_{ij}^2, n_{ij}^1)$ is the tangential direction and $\Delta v_{ji}^\tau = (\vec{v}_j - \vec{v}_i) \cdot \vec{t}_{ij}$ is the tangential velocity difference. The function $g(x)$ is zero if $x < 0$ and otherwise is equal to x.

The interaction force $\vec{f}_{iW}$ is specified analogously [5]:

$$\vec{f}_{iW} = A_i \exp\big[(r_i - d_{iW})/B_i\big]\vec{n}_{ij} z(\cos\varphi_{iW}) z(R_p - d_{iW}) + K g(r_{iW} - d_{iW})\vec{n}_{iW} + \kappa g(r_{iW} - d_{iW})(\vec{v}_i \cdot \vec{t}_{iW})\vec{t}_{iW} \tag{7}$$

where d_{iW} denotes the distance to the wall W, $\vec{n}_{iW}$ is the direction perpendicular to the wall W, $\vec{t}_{iW}$ is the direction tangential to it, φ_{iW} is the angle between the direction of $\vec{v}_i$ and the direction $-\vec{n}_{iW}$ and $z(\cos\varphi_{iW})z(R_p - d_{iW})$ are used to indicate whether the wall W can be spotted by pedestrian i.

In this paper, the model parameters for the standard social force model are adopted as in Ref. [5] $A_i = 2 \times 10^3$ N, $B_i = 0.08$ m, $\tau = 0.5$ s, $K = 1.2 \times 10^5$ kg s^{-2}, $\kappa = 2.4 \times 10^5$ kg m^{-1} s^{-1}.

2.3. Determination of $\vec{e}_i(t)$

In our model, it is assumed that there are two types of pedestrians: Informed individuals know the exit location whereas uninformed individuals do not. The desired escape directions for informed and uninformed individuals are determined differently. For an informed individual i, his/her desired escape direction $\vec{e}_i(t)$ is determined by his/her memory about the exit location, the vector $\vec{e}_i(t)$ is given by:

$$\vec{e}_i(t) = \vec{p}_M \tag{8}$$

where $\vec{p}_M = (E - P)$ is the direction information from informed pedestrian's memory. Here $P = (P_x, P_y)$ is the pedestrian's current position, and $E = (E_x, E_y)$ is location of exit.

For an uninformed individual, his/her decision-making for desired escape direction is associated with four way-finding behaviors, i.e., wall-following behavior, communication with informed individuals, memory information obtained from communication, and herding behavior. Two cases are classified for an uninformed individual i: if there is at least one informed individual within his/her perceptual field, the desired escape direction $\vec{e}_i(t)$ is given by:

$$\vec{e}_i(t) = c_w\vec{p}_w + c_n\vec{p}_n + c_m\vec{p}_m \tag{9}$$

otherwise

$$\vec{e}_i(t) = c_w\vec{p}_w + c_m\vec{p}_m + c_h\vec{p}_h \tag{10}$$

where $\vec{p}_w$ represents the direction information obtained from wall-following behavior, $\vec{p}_n$ is the direction information obtained from communication, $\vec{p}_m$ is the direction information from memory. We will address $\vec{p}_w$, $\vec{p}_n$ and $\vec{p}_m$ in sections 2.4 and 2.5. $\vec{p}_h$ represents the herding direction and is given by the normalized average moving direction $Norm(\langle v_j(t)\rangle_i)$ of all other pedestrians j [5] within his/her visual field. If there is no pedestrian within his/her visual field, $\vec{p}_h$ will be set to zero. c_w, c_m, c_n, c_h are weight coefficients.

For the situation that an uninformed individual is far from the wall and far from any other individuals, he/she will keep moving at his/her direction until he/she meets someone or sees the wall. Finally, when an uninformed individual sees the exit in his/her visual field, he/she will behave as an informed individual and the vector $\vec{e}_i(t)$ will be given by formula (8).

In the simulations of section 3, we use the parameters $c_w = 1/R_p$ indicating that pedestrians are closer to the wall in a narrower view condition. Other three parameters are chosen as $c_m = 0.8$, $c_n = 0.5$, $c_h = 0.2$. We have tested a wide range of these coefficients and found that they have no qualitative effect on the results.

2.4. Wall-following behavior

Wall-following behavior is often observed during people's way-finding process, in particular when the exits are invisible [5,16]. In our model, the wall-following behavior is considered only when the wall is within the visual field of an uninformed individual and the exit is out of his/her visual field, see Fig. 2. We denote the intersections of his/her visual edge and the wall as the attractive points. For pedestrians i and j shown in Fig. 2, the attractive points are A_i^+ and A_i^-, A_j^+ and A_j^-, respectively. Then the pedestrians will decide which direction to go. Take pedestrian i for example, if he/she tends to follow and explore the wall anticlockwise (+), $O_iA_i^+$ is selected as the attractive direction, and $\vec{p}_w$ is then defined as $\vec{p}_w = Norm(O_iA_i^+)$; otherwise, if he/she tends to explore the wall along the clockwise direction (−), $O_iA_i^-$ is selected as the attractive direction, and $\vec{p}_w$ is then defined as $\vec{p}_w = Norm(O_iA_i^-)$.

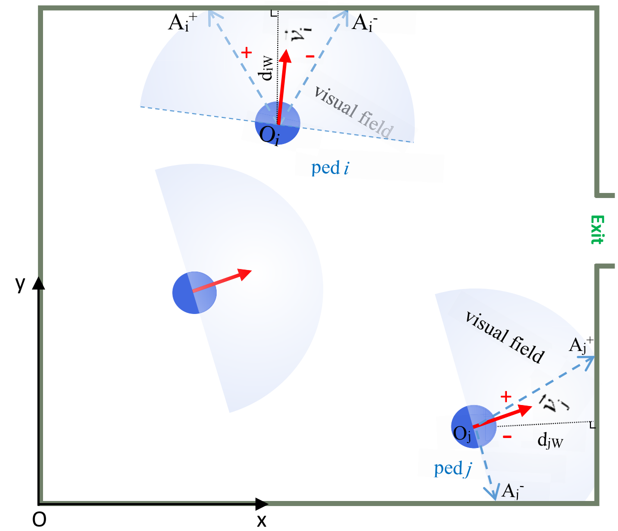

Fig. 2. Illustration for wall-following mechanism.

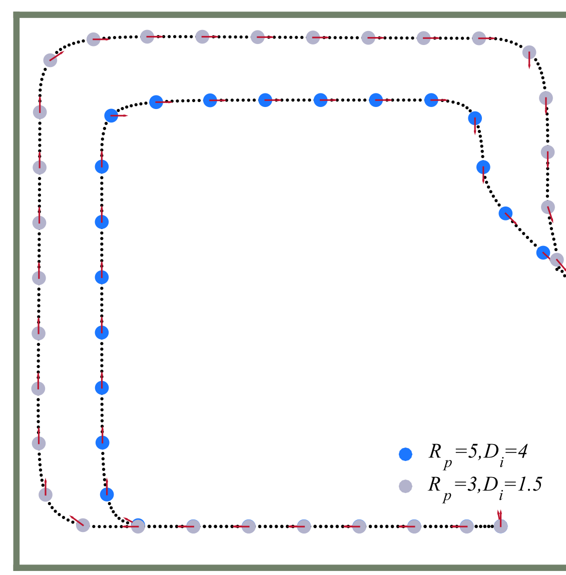

Fig. 3. Simulation results of the trajectories of wall-following behavior under two different sets of parameter values. (The dotted lines are the trajectories, the filled circles denote pedestrian's position at each second, the red arrows denote the desired moving direction at that point.)

The wall-following direction (clockwise or anticlockwise) that the pedestrian will adopt depends on the situation when the pedestrian spots the wall for the first time. For example, assuming that pedestrian j sees the wall for the first time as shown in Fig. 2. The attractive direction is given by the one which has a smaller angle with his/her current walking direction. Then $O_jA_j^+$ is selected (angle $\langle O_jA_j^+, \vec{v}_j\rangle$ is smaller than angle $\langle O_jA_j^-, \vec{v}_j\rangle$) and his/her wall-following direction will be set as "+" (exploring the wall anticlockwise).

Finally, when the distance to the wall d_{iW} is below a threshold D_i, we assume that pedestrian i's wall-exploring direction will be parallel to the wall. Here, we set $D_i = \lambda_i R_p$ with λ_i being a uniformly distributed random number between 0.5 and 1.

Fig. 3 displayed two example trajectories of the wall-following behavior based on the mechanism described above. One can see that it can well depict the wall-following behavior.

2.5. Communication and memory

We assume that (i) uninformed individuals always ask pedestrians within their perceptual field that "who knows where the exit is"; (ii) the informed individuals always answer if they hear the voices of uninformed ones. For an uninformed individual i, we denote the nearest informed individual within his/her perceptual field as j. We suppose i obtains information only from communication with j.

The direction information obtained via communication is given by $\vec{p}_n = (N_c - P)$. Here $N_c = (E_x^j + \eta_x, E_y^j + \eta_y)$, (E_x^j, E_y^j) is exit location that j knows, η_x, η_y are noises to describe the uncertainty during information transfer in the communication. In the simulations we use $\eta_x \sim N(0, 0.5)$ and $\eta_y \sim N(0, 0.5)$, where $N(\mu, \sigma^2)$ denotes a Gaussian distribution with mean μ and variance σ^2. $P = (P_x, P_y)$ is pedestrian i's current position.

When an uninformed individual obtains the exit location from communication, he/she can memorize the location. Thus, the memory information is given by $\vec{p}_m = (N_c - P)$. When new exit location information is obtained by communication, we suppose the uninformed individuals update the memory information with a probability α. A higher value of α indicates that individuals are more susceptible to the new information and in the simulations we use $\alpha = 0.5$.

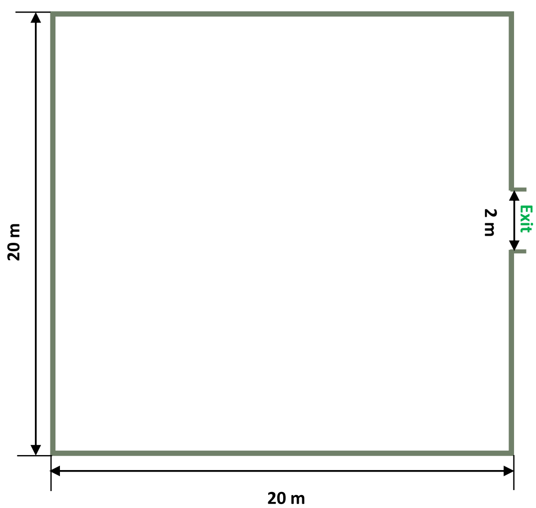

Fig. 4. Example of the evacuation space.

3. Simulation and results

The evacuation space is defined as a 20 m × 20 m square hall with only one 2 m-wide exit located in the center of the right-side wall as shown in Fig. 4. Initially the pedestrians are distributed randomly with random velocities in the space. The desired speed is set as $v^0 = 2$ ms^{-1}, and the maximum speed for each pedestrian $v^{max} = 3$ ms^{-1}. The exit location is $M_e = (20, 10)$ in the middle of the exit.

3.1. Influence of communication

In this section, we investigate the influence of communication on the evacuation process. We perform simulations in cases with and without communication. Figs. 5 and 6 show the snapshots of the two cases, with a total number of 80 pedestrians and 10% of them being the informed ones (denoted by green filled circles in the figures). The radius of perceptual field of each pedestrian is set to 3 m. When uninformed individuals are able to communicate, one can see from Fig. 5 that most of them gather to the exit

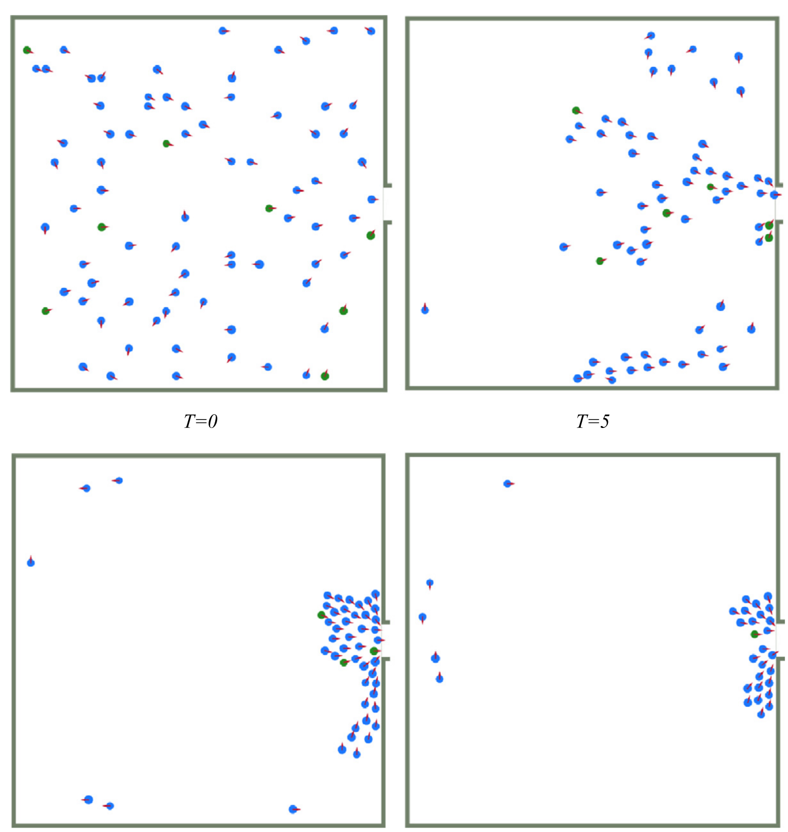

Fig. 5. Simulation snapshots for the case with communication at different times (seconds).

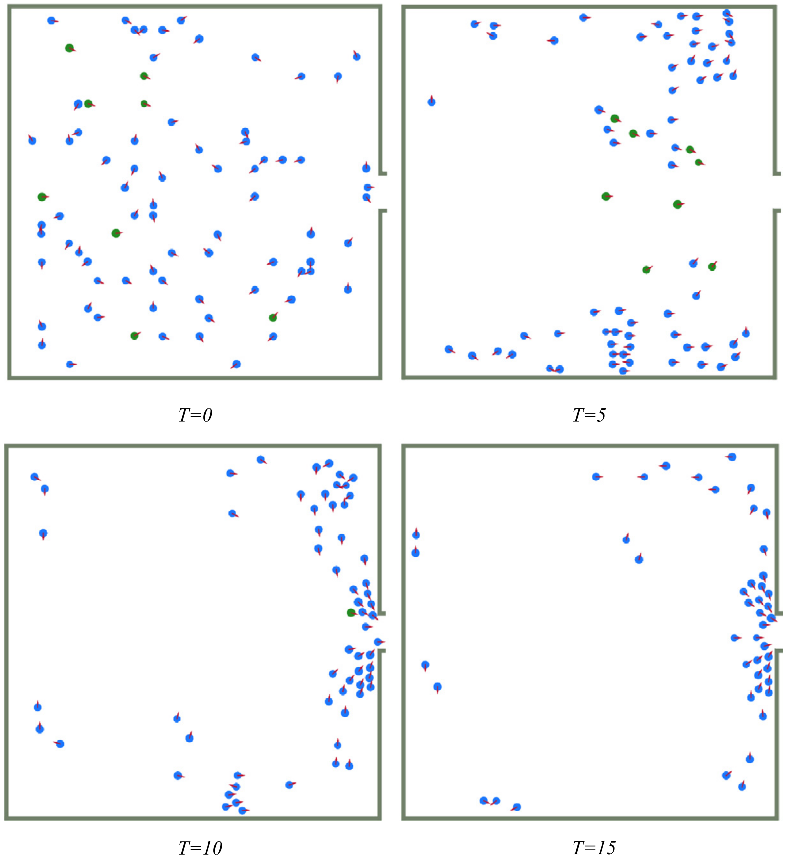

Fig. 6. Simulation snapshots for the case without communication at different times (seconds).

quickly and their desired directions exhibit a collective consensus. In contrast, if they are unable to communicate, the individuals distribute more scattered and most of these individuals find the way out via the wall-following behavior and herding behavior, see Fig. 6.

Figs. 7 and 8 show the evacuation time differences between the cases without and with communication under $R_p = 3$ m and $R_p = 2$ m, respectively. As expected, the evacuation times in the case with communication are always less than that without communication. Moreover, we find that the increase of the percentage of informed individuals will lead to an overall decrease of the time differences. When uninformed individuals are able to communicate with others, communication and memory effect will guide them to find the exit quickly. In contrast, in the case without communication they will spend much more time to find the exit. This difference expands with the increase of the size of the crowd when percentages of the informed individuals are low, as shown by the lower parts in Fig. 7 and Fig. 8. However, when the percentages of informed individuals increase, the evacuation times of larger sizes of the crowd will mainly depend on the capacity of the exit because congestion occurs quickly in the vicinity of the exit. Thus, the time differences between the two cases shrink, as shown by the top right regions in Fig. 7 and Fig. 8. Moreover, Fig. 7 and 8 also show that the time differences increase when R_p decreases from 3 m to 2 m, which indicates that communication has better benefit in a more adverse environment.

Figs. 9 and 10 compare evacuation times in the cases with and without memory effect under $R_p = 2$ m and $R_p = 3$ m, respectively. One can see that in general the memory effect will facilitate the evacuation process, and the differences between the two cases reduce when the radius of the perceptual field increases, see Fig. 10.

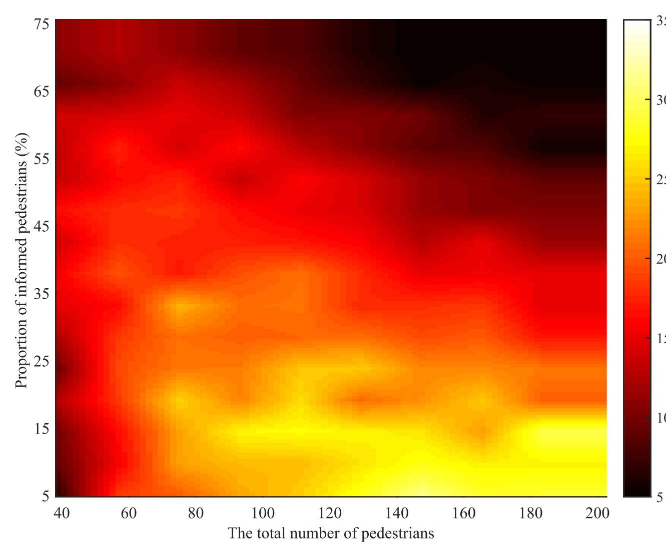

Fig. 7. Evacuation time differences of the two cases under $R_p = 3$ m.

3.2. Emergency exit

Now, we investigate how communication affects the efficiency of an emergency exit. We perform a series of simulations in a hall with two 2 m-wide exits. The right-side exit is the main exit (denoted by A) and the left-side one is an emergency exit (denoted by B), see Fig. 11. Initially, 100 pedestrians are distributed randomly in the hall. It should be noted that, for simplicity, each informed individual is assumed to know only one exit, either exit A or exit B. The exit choice behavior has not been taken into account [24–26]. However, for the uninformed individuals, their exit

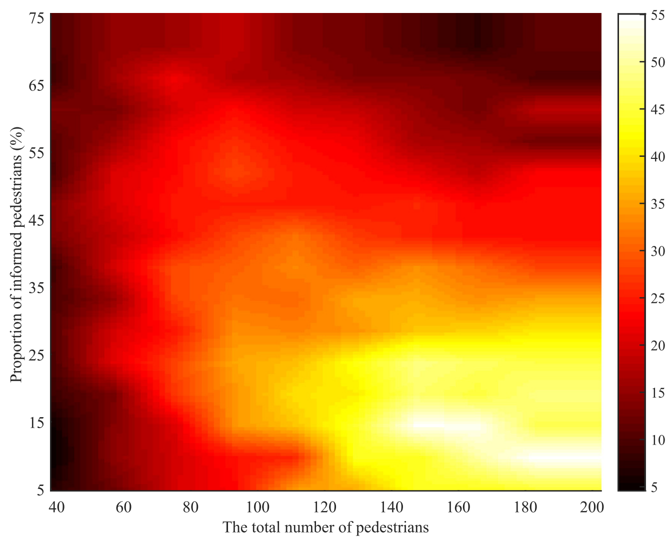

Fig. 8. Evacuation time differences of the two cases under $R_p = 2$ m.

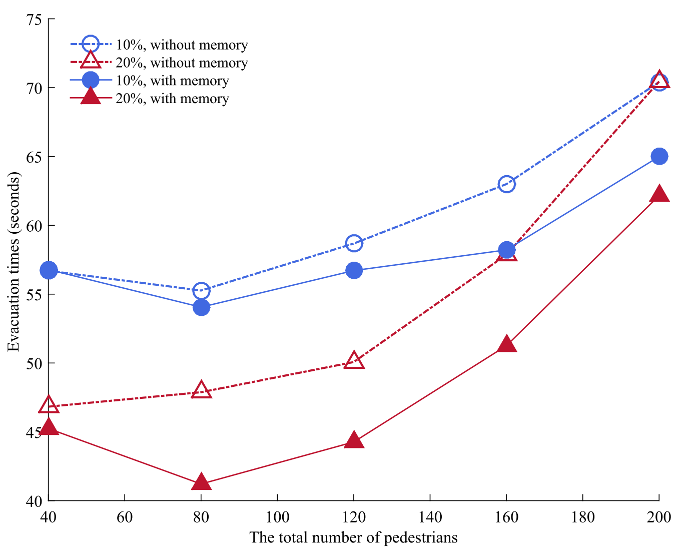

Fig. 9. Comparison of the evacuation times with and without communication under different percentages of informed pedestrians. The radius of the perceptual field $R_p = 2$ m.

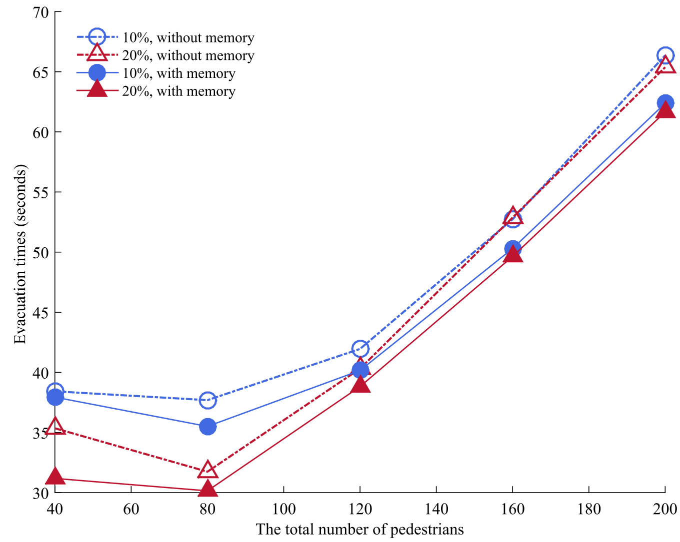

Fig. 10. Comparison of the evacuation times with and without communication under different percentages of informed pedestrians. The radius of the perceptual field $R_p = 3$ m.

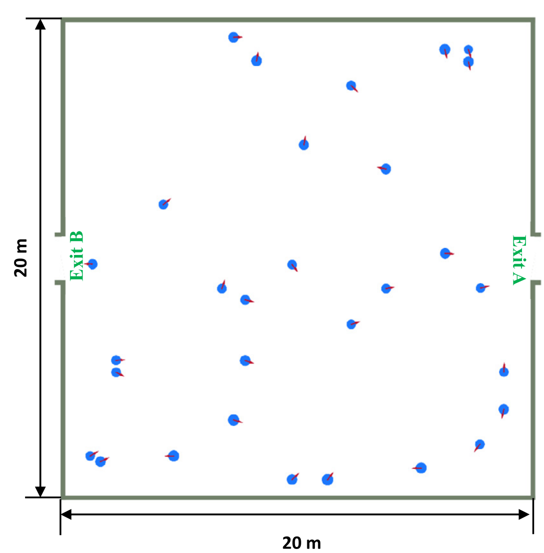

Fig. 11. Evacuation space with a main exit A and an emergency exit B.

choice may change in the evacuation process due to communication with different informed individuals.

Firstly, we assume that there are 10% informed individuals, who only know the main exit A. We perform 100 runs of simulation for cases with and without communication. It takes averagely about 29 seconds for all the pedestrians to evacuate in the former case, while it takes about 33 seconds in the latter case. In the former case, uninformed individuals are able to communicate. Thus, the information about exit A becomes dominant during the evacuation process. One can see from Fig. 12 that most of pedestrians leave through exit A. However, in the latter case, pedestrians are more likely to find the emergency exit B since they are not able to communicate with others. The utilization of emergency exit B has increased.

Next, we assume that a certain number of individuals only know about the emergency exit B. The number of individuals who only knows about the main exit A is assumed to remain unchanged (10%). From Fig. 13, we can see that the increase of the proportion of individuals knowing exit B will lead to an increase of the number of evacuees leaving through the emergency exit B. The evacuation time (averaged over 100 replications) reduces from 28 seconds to 21 seconds, see Fig. 14.

4. Conclusion

This paper studies pedestrian evacuation in view and hearing limited condition based on the social force approach. It is assumed that there are two types of pedestrians in the evacuation process. Informed individuals know the exit location. Their desired escape directions are determined by their own memory about the exit location. In contrast, uninformed individuals do not know the exit location. However, they can communicate with the informed ones within their perceptual fields, thus learn to know and memorize the exit location. The decision-making of an uninformed individual is assumed to be associated with wall-following behavior, communication with informed individuals, memory information obtained from communication, and herding behavior.

Our simulations show that if uninformed individuals are able to communicate and memorize, the evacuation efficiency can be enhanced. The enhancement depends on factors such as propor-

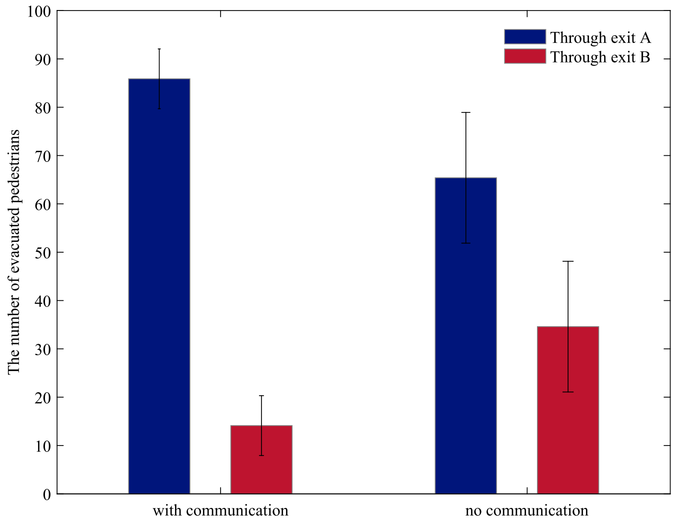

Fig. 12. Utilization of two exits in the case of communication and no communication.

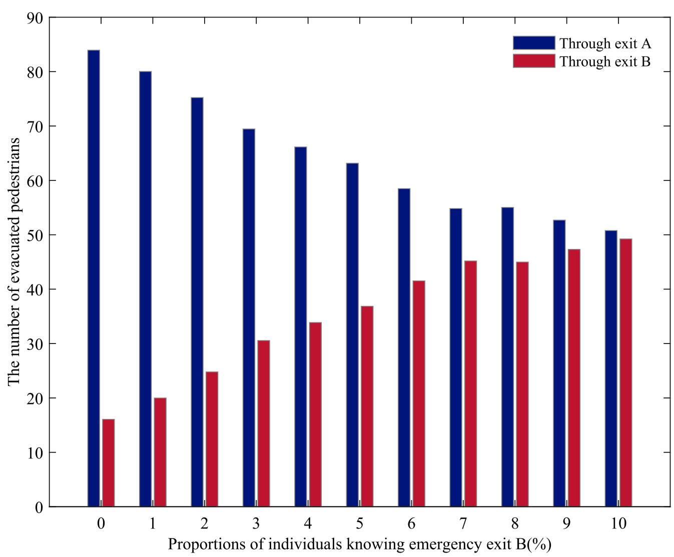

Fig. 13. Utilization of two exits vs the number of individuals knowing exit B.

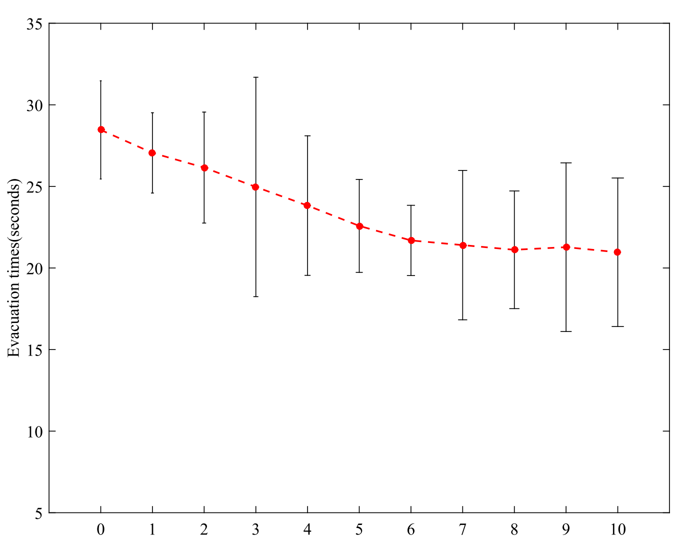

Fig. 14. Mean evacuation time and standard error.

tion of informed individuals, initial density of pedestrians, range of the perceptual field. When the pedestrian density is large and congestion quickly emerges in the vicinity of the exit, the effect of communication and memory seems not significant. Moreover, communication has better benefit in a more adverse environment.

Finally, we would like to mention that for simplicity, we have assumed that each informed individual only knows one exit in the multiple-exit situation. In our future work, the exit choice needs to be taken into account. Note that different from previous studies of route and exit choice in evacuation process, the pedestrians do not know the complete information of the scenario since the hearing and view are limited. How to model the exit choice behavior realistically under such circumstance might be a tough issue.

Acknowledgement

This work is financially supported by the National Basic Research Program of China (No. 2012CB725400), China National Science Funds for Excellent Young Researchers (71222101), and the National Natural Science Foundation of China (71210001, 71471012).

Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physleta.2016.07.030.

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