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Private financing and mobility management of road network with tradable credits

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A B S T R A C T

Tradable credits have been recognized as a powerful instrument and are increasing used in many fields. This paper employs the tradable credits scheme on traffic mobility management and private provision of public transportation infrastructure through a novel kind of private financing of public road: build-equity-credit (BEC) scheme, hoping to achieve a triple win. Namely, the government can achieve its objectives (e.g. construction of the new road, desired traffic condition, certain vehicle emissions threshold) without its own capital, the private firm can receive its expected profit with less public's resistance and the travelers can enjoy less congested traffic with a negligible cost. Moreover, many issues that occurred upon the termination of the traditional private financing (e.g. build-operate-transfer) scheme, such as severe congestion, explosion of travel demand and lack of management and maintenance, can be avoided in BEC. A general bi-level programming problem is formulated to model the determination of capacity of the new road and the tradable credits scheme in BEC scheme. The properties of several different BEC scenarios are investigated. Generally, the link service level in BEC is not constant but depends on multiple factors. Under some conditions, the total market value of the credits charged on the new link can offset its construction cost and the profit of the private firm can always be nonnegative.

1. Introduction

Over the past decades, the participation of the private sector in transport infrastructure services has become popular worldwide, which rebalances the roles of public and private sectors played in public service delivery. Such partnerships between the public sectors and the private sectors are often known as public-private partnership (PPP). The interest in PPP is partly driven by the fact that the required huge amount of investment for providing and maintaining transportation infrastructures has imposed a great challenge to the governments worldwide. The increasingly tight government budgets further aggravate this situation. The government, therefore, is in favor of having the private investors to play an increased role in the investment and development of transport infrastructure. Privatization is further supported by the popular view that private firms operate more efficiently because of their profit motivation, thereby lowering the cost of construction and providing better service level (Yang and Meng, 2000).

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Generally, PPP is arrangements typified by joint working between the public and private sectors for a long term (HM Treasury and OGC, 2005). Based on different combination of services, roles and responsibilities and different financing methods between the government and the private sectors, there are multiple alternatives for PPP. Mu (2008) synthesized the alternatives of PPP, which includes many different forms: design-bid-build (DBB), service contract, design-build (DB), design-build-operate (DBO), design-build-operate-maintain (DBOM), build-operate-transfer (BOT), build-operate-own (BOO), design-build-finance-operate (DBFO). A two-dimension framework (Fig. 1) is used to depict these alternatives. The horizontal axis stands for the continuum of delivery methods measured by the degree to which the elements (design, financing, construction, operation, maintenance) of PPP projects are unbundled or bundled with one another, while the vertical axis stands for the continuum of financing method measured by the degree to which public sector or private sector undertakes the financing responsibility.

Build-operate-transfer (BOT) is one of the most popular options of PPP, where a private firm builds roads at its own expense. It then operates and charges the travelers who use the roads during a specified period to recover cost and/or receive profits and finally, these roads will be transferred to the government. There are many benefits of using BOT for transport infrastructure investment. For example, as mentioned above, it is widely believed that the private provision of roads can provide more efficient operation and management of transportation facilities. Also, the public sector, facing taxpayer resistance, may simply be unable to finance facilities that the private sector would be willing and able to undertake for a profit. Furthermore, almost all users can benefit from the BOT project, even those who do not use these new roads would benefit from reduced congestion on the old ones (Yang and Meng, 2000).

Given the advantages of BOT, it has attracted fast-growing interest in both theory and practice. Among the existing literature about BOT, many analyses have focused on the capacity choices and tolls setting as well as the profitability and social welfare gain (e.g. Yang and Meng, 2000; Lindsey and Verhoef, 2001; Yang and Huang, 2005; Lindsey, 2006) and also, properties of link service level and self-financing have been well investigated. It has been proved that, under certain assumptions, the link service level in terms of the volume-to-capacity ratio, offered by a profit-maximizing private firm on a private road is constantly equal to that under social optimum (Xiao et al., 2007; Yang et al., 2009; Tan et al., 2010; Wu et al., 2011; Wang et al., 2013; Niu and Zhang, 2013). Yang and Meng (2002) showed that self-financing holds for general network under some conditions.

While being popular, BOT scheme suffers from a few social, political and technical issues. Evidences show that many BOT projects in China continue to charge tolls for many years after the concession period is ended. This has caused the public's questioning of corruptions and has met great objection and criticism although many firms argue that the tolls are used for further management and maintenance (e.g., Zhang, 2014; Xiao and Cao, 2014). However, if the toll charge is ceased, questions emerge, such as lack of maintenance and management, enormous congestion, and explosion of travel demand (Gu, 2013). For example, the Capital Airport Expressway in Beijing attracted forty percent more traffic flow after it stopped charging drivers, which led to heavy traffic congestion almost from 6 a.m. until 6 p.m. (Deng, 2011). It is impossible to recharge the road now for social and political reasons but leaving it alone results in great efficiency losses. Furthermore, to be equitable, the construction costs of the new road should be borne by all those who benefit from the increased traffic efficiency. However, as mentioned above, although all travelers can benefit from the BOT project, the construction and maintenance costs are only borne by travelers who use the new transport infrastructure. This may be unfair for travelers who bear a significant part of the huge construction costs by paying the toll charge, especially for the travelers who have no alternative roads. This is also one of the major reasons that the BOT scheme is rejected by the public.

Another concern with BOT project is that the government will lose control of the new transportation infrastructure during the concession period under operations of the private firm. The government thus faces difficulty in integrated planning, management and operations of the whole transportation infrastructure system. It is also very difficult for the government to deal with some unexpected situations. A noteworthy example is the Fifth Ring Road in Beijing. When the Fifth Ring Road in Beijing was completed and tolled in 2003, the average flow was only 200–300 veh/h while its capacity was 3000 veh/h. The usage of the road and the resulting daily toll revenue was far below the planners' estimation, leading to a very embarrassing situation. Eventually, the government had no choice but ceased the tolling. However, because the Fifth Ring Road was a BOT project engaged by a listed company, the government had to buy back the project for full control. As a result, all parties suffered great losses (Zhang, 2004).

In view of these issues, this study proposes and examines a new PPP alternative: build-equity-credit (BEC), where tradable credits are employed to substitute the road tolls. BEC is also a form of commercial and private provision of transportation infrastructure. A private firm builds the new road at its own expense. However, different from BOT, the private firm does not operate the new road with toll charge but instead is granted some equities of the whole road network system as a reward for its construction. At the same time, the government will operate the road as well as the whole road network system by designing a new or redesigning an existing tradable credit distribution and charging scheme for achieving a predefined quantitative objective (e.g. social welfare maximization, a certain vehicle emissions threshold control). As a stockholder, the private firm will receive a given amount of tradable credits every period. The rest of the credits will be distributed to all the eligible travelers. Travelers pay credits to use the roads and can freely trade credits in the credit market. The private firm sells its credits in the credit market to gain profit. Therefore, every traveler in the network, regardless of their usage of the new roads, will undertake a minor part of the construction costs. After the private firm gains expected profit, the equities it owns will be transferred to the government but the tradable credit distribution and charging scheme will continue to prevent severe congestion and explosion of travel demand. The government can also keep some credits to sell in the market to

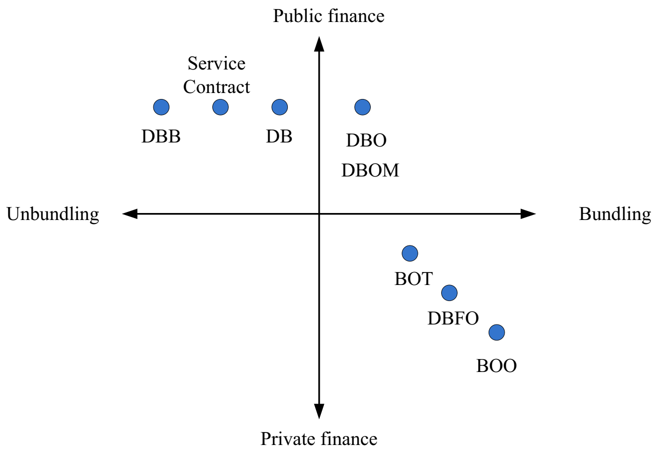

Fig. 1. Two-dimension framework of PPP options (Mu, 2008).

get the fund for the road management and maintenance afterwards. Because the total number of travelers in the network is sufficiently large, every traveler only needs to bear a negligible part of the construction costs and can thus make the introduction of the BEC scheme more acceptable to the public. Furthermore, the government can keep its control of the new road and make overall traffic plans to achieve desired goals; the private firm can receive its expected profit without the public's rejection.

Since tradable credits/permits were first investigated by Crocker (1966), Dales (1968) and Montgomery (1972), the typical view has been that "in terms of production efficiency, the tradable quota system is equivalent to that of the Pigouvian tax" (Sandmo, 2000). Furthermore, the government can assign the credits just as it pleases because the initial distribution does not affect the equilibrium allocation in a perfect market. In other words, the tradable credits scheme appears to combine the concern for efficiency with the government's concern for redistribution (Harstad and Eskeland, 2010), which makes its introduction been recognized as a powerful instrument in many fields. In the environmental field, particularly in relation to water use, fishing and pollution, the tradable credits are increasing employed in practice (including, for example, the European Union Emission Trading Scheme (Perrels, 2010)). In mobility management, the tradable credits scheme is considered as a much more equitable and acceptable measure than road toll pricing. It has received much attention in recent years after Yang and Wang (2011) formally set up the mathematical model.

The tradable credits scheme in mobility management has a number of characteristics: (1) the government issues a predetermined amount of credits periodically; (2) the credits are, at least for the most part, distributed for free; (3) travelers must pay credits to use road-links; (4) the credits can be traded freely in a competitive market without government intervention. In this paper, we employ the tradable credits, on one hand, to raise the revenue for the construction of the new road (the profit of the private firm), on the other hand, to control the traffic demand and travelers' route choices. We hope the use of the tradable credits can help the BEC scheme to be triple win, namely, the government can achieve its objectives (e.g. construction of the new road, desired flow pattern, certain vehicle emissions threshold) without its own capital, the private firm can receive its expected profit with less resistance and the travelers can enjoy less congested traffic with a negligible cost.

The focus of this paper is twofold. Firstly, the paper aims to investigate the design of the tradable credits scheme and the capacity choice of the new road in BEC. Selection of the capacity of the new road and design of the tradable credits scheme of the whole network will greatly affect the relevant benefits of the private investor, the road users and the society. For instance, the capacity of the new road will affect the investment level, the traffic demand and further the social welfare. The tradable credits scheme of the whole network will affect the flow pattern, the travel demand, the price of the credit and the net profit of the private firm. All these are the main factors that the planner must consider in the construction of the new road. Thus, it is essential to jointly study the determination of the capacity of the new road and the tradable credits scheme of the whole network.

The second aim of the paper is to investigate the properties of the models in BEC. Generally speaking, the link service level indicates the congestion level of the road and the delay of the travelers. Therefore, it is significant to investigate the properties of the link service level in BEC and its influential factors. The results show that the link service level depends on multiple influential factors including the travel time function, the construction cost, the marginal travel time and the marginal travel cost. The profit of the private firm or the self-financing of the new road in the private provision, especially in the social welfare maximization is also very important. We proved that under certain conditions, the total market value of the credits charged on the new link could offset its construction cost and the profit of the private firm can always be nonnegative in the social welfare maximization in BEC.

The rest of this paper is organized as follows. BEC is illustrated in detail in Section 2. In Section 3, a general bi-level programming model is introduced to determine the optimal choice of capacity for a particular road and the tradable credits scheme for the whole network in BEC. Sections 4 and 5 investigate several different BEC models under two typical kinds of tradable credits scheme, respectively. Section 6 provides discussion of the properties and policy implications of different BEC models. Section 7 gives a numerical example and Section 8 draws the conclusions.

2. Build-equity-credit scheme

The BEC scheme proposed here is a combination of private financing and mobility management, where a private firm finances and constructs a new road and gains equities of the road network system, and the government designs a tradable credit distribution and charging scheme to pay the construction costs and manage mobility.

The process of BEC (in Fig. 2) can be divided into three periods as follows. In the first period, a private firm builds a new road at its own expense. The new road together with the existing roads constitutes a new road network. In the second period, the government operates the new road network system and gives a part of the equities of the road network system to the private firm as the reward of constructing the road. The government then designs a distribution and charging tradable credits scheme. The private firm will get some credits for free according to the equities it owns. The other credits are distributed to all eligible travelers. Travelers must pay credits to use roads based on the credit charging scheme. The private firm can sell its share of credits to travelers to receive profit. After the private firm gets its expected profit, the second period is over and the equities owned by the private firm will be transferred to the government. In the third period, the government can redesign the credit distribution scheme, where all the credits are distributed to travelers or the government can keep a small part of credits to get the funds for maintenance and management of the road network afterwards. The credit charging scheme will continue to manage mobility and sustain the desired traffic flow pattern.

(1) The equities assignment between the government and the private firm. In BEC, the private firm constructs the new road at its own expense. In return, it gets a small part of equities of the road network. Most of the equities of the road network are still owned by the government so that it can control the road network system. The private firm's share of equities depends on the traffic flow of the new road and its construction and maintenance costs.
(2) The tradable credit distribution scheme. To operate the road network system, the government can issue and charge tradable credits every period (for example, a month). As a stockholder, the private firm can possess a certain ratio of tradable credits, which is determined by the share of equities it owns or the total credits charged on the new road. The remaining credits are distributed to eligible travelers for free. The distribution of credits between travelers can be either Origin-Destination (OD) based distribution or uniform distribution. After receiving its expected profit, the private firm will not obtain free credits anymore. The government can keep a small part of credits every period to obtain the funds for maintenance and management of the road network afterwards. All the other credits are still distributed to travelers for free.
(3) The tradable credit charging scheme. The government designs a tradable credits charging scheme to manage mobility and achieve desired goals, such as social welfare maximization and a certain vehicle emissions threshold. The credit charge is link-specific. Travelers must pay credits to use roads based on the credit charging scheme. Depending on their origin, destination and route choices, travelers' consumption of credits for traveling will in general differ. They can buy/sell extra credits in the credit market according to their travel needs. The private firm can also sell its credits in the market to get profit.

BEC is a new kind of PPP, which is very similar to BOT. Fig. 3 further illustrates the commonalities and differences between BEC and BOT. From Fig. 2 and Fig. 3, BEC and BOT are in common in that the private firm finances and builds the new road and gains profit from this process. Their differences lie in the operation of the new road, the way the private firm gets its profit and the mobility management of the road network. In BOT, the private firm operates the new road and obtains its profit by directly charging travelers who use the new road during the concession period. In contrast, in BEC, the government manages and operates the whole road network system including the new road, and the private firm gets a part of the equities of the whole road network system and receives its profit by selling credits in the market. In other words, the private firm owns and operates the new road in BOT while it holds equities of the road network system in BEC. Furthermore, BEC incorporates the tradable credits scheme to manage mobility, where the government can design a tradable credit distribution and charging scheme to achieve desired traffic flow pattern. However, in BOT, the government can only take measures on the old roads in the network and thus is incapable of managing the whole network system.

The BEC scheme can achieve a triple win. From the perspective of the government, the new road is constructed with private fund. Secondly, the government still retains the integrated operation of the entire road network through an overall mobility management scheme to achieve desired goals. Finally, it avoids the problems caused by the termination of the toll charge, for example, it can still obtain the funds for the subsequent maintenance and management of the roads and implement congestion control by the ongoing mobility management scheme. From the perspective of the private sector, the private firm can lower the risk in its private undertaking. Firstly, it reduces the public's objection and resistance because no tolls are charged; secondly, it avoids the situation that the traffic flow on the new road is too low that the private firm cannot get expected profit. From the perspective of the users, travelers benefit from the new road and the mobility management scheme introduced. Firstly, the traffic conditions are improved as a result of the new add-on road; secondly, it is more equitable as

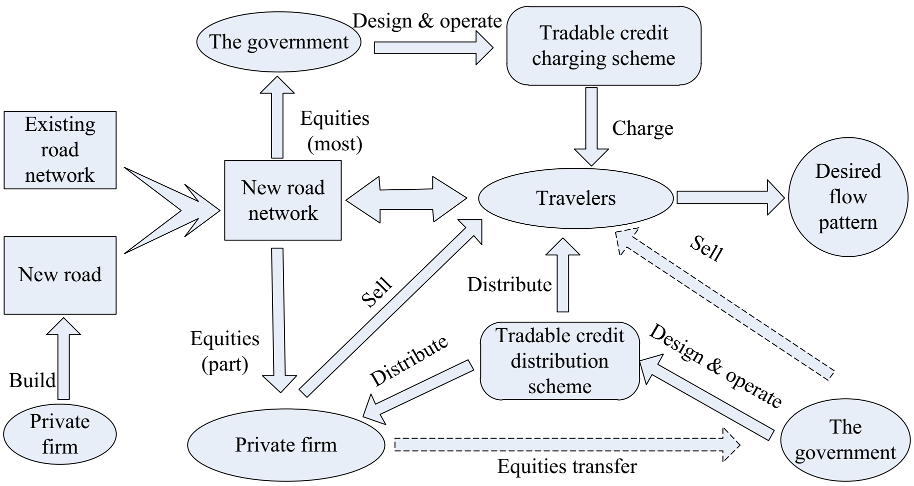

Fig. 2. The flowchart of BEC.

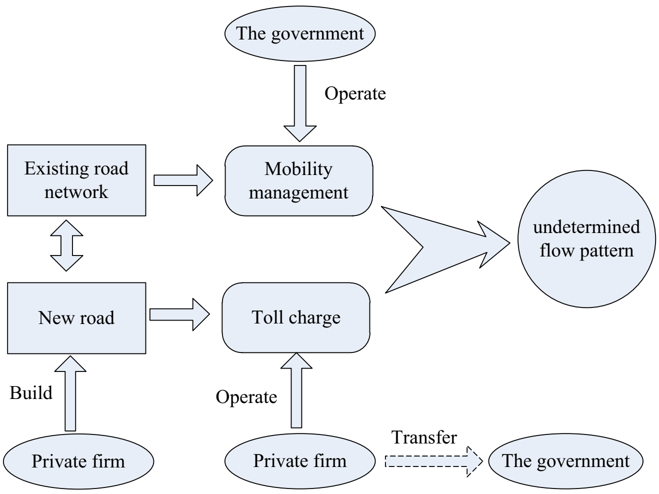

Fig. 3. The flowchart of BOT and the mobility management of the government.

every traveler pays for the benefit he/she gains from the new road; finally, as the total number of travelers in the whole road network is large, the construction and maintenance costs each traveler bears is trivial.

3. Model formulation

Consider a transportation network $G = (N, A)$ with a set N of nodes and a set A of directed links. Each link $a \in A$ has a separable link travel time function, $t_a(v_a)$, which is assumed to be nonnegative, differentiable, convex and monotonically increasing with its flow v_a . Let $f_{r,w}$ denote the route flow on route $r \in R_w$ for OD pair $w \in W$ and Ω represent the set of feasible flow pattern defined by

$$\Omega = \left\{ (v, q) | v_a = \sum_{w \in W} \sum_{r \in R_w} f_{r,w} \delta_{a,r}^w, q_w = \sum_{r \in R_w} f_{r,w}, f_{r,w} \geqslant 0, 0 \leqslant q_w \leqslant q_w^{\max}, r \in R_w, w \in W, a \in A \right\} \tag{1}$$

where $\delta_{a,r}^w = 1$ if link a is on route r , and 0 otherwise. q_w is the travel demand between OD pair $w \in W$ and $q_w^{\max}$ is the potential or maximum demand between OD pair $w \in W$. Let y_l denote the capacity of the new link l. The link construction cost function is denoted as $I(y_l)$ and is assumed to be an increasing and differentiable function of link capacity.

When a BEC contract is signed, a decision in terms of a combination of the tradable credits scheme and the road capacity must be made with the consideration of the social welfare and the profit of the private firm. The social welfare and the profit of the private firm are related to the decision variables through the travel demand and the traffic flow pattern. This relationship can be modeled as a bi-level programming problem. Assume that the government wants to build a single new link l in a general network with the BEC project. A predetermined amount of credits φ_w is distributed to all travelers between OD pair $w \in W$. The total amount of credits issued by the government is denoted as K and satisfies $\sum_{w \in W} \varphi_w q_w + \gamma \mathrm{K} = \mathrm{K}$, where γ denote the credit ratio distributed to the private firm. Travelers must pay credits to use road links based on a nonnegative credit charging scheme $\kappa = (\kappa_a \geqslant 0, a \in A)$. They can also trade credits in the credit market. Then, the general BEC problem can be formulated as the following bi-level programming problem:

$$\max_{\kappa, y_l} F(\kappa, y_l, v, q) \tag{2}$$

subject to:

$$G(\kappa, y_l, v, q) \leqslant 0 \tag{3}$$

where $(v, q)$are obtained by solving

$$\min_{(v,q)\in\Omega} \sum_{a \in A} \int_0^{v_a} t_a(\omega) d\omega - \sum_{w \in W} \int_0^{q_w} B_w(\omega) d\omega \tag{4}$$

subject to:

$$\sum_{a \in A} \kappa_a v_a \leqslant \mathrm{K} \tag{5}$$

Eq. (5) is a network-wide credit feasibility condition, which means the total amount of credits needed for all travelers going through the network must be not larger than the total amount of credits issued. Under a given tradable credits scheme and a given capacity of the new link, the user equilibrium (UE) flow pattern with elastic demand can be found from the above lower-level UE problem (4) and (5).

$F(\kappa, y_l, v, q)$ and $G(\kappa, y_l, v, q)$ are the objective function and the constraint function in BEC, respectively. Generally speaking, the objective function of the upper-level problem of the BEC project is a trade-off between the social welfare and the profit of the private firm. However, for clarity and simplicity, here we just consider the two extreme cases, i.e., the social welfare or the profit of the private firm. The social welfare depends on the credit scheme (the credit charge of the new link) and the size of the project (the capacity of the new link). It can be defined as "the sum of consumers' and producers' surplus" (Yang and Meng, 2000):

$$Z(\kappa, y_l, v, q) = \sum_{w \in W} \int_0^{q_w} B_w(\omega) d\omega - \sum_{a \in A, a \neq l} t_a(v_a) v_a - t_l(v_l, y_l) v_l - \alpha I(y_l) \tag{6}$$

The profit of the private firm equals the revenue it gains minus the construction cost. The construction cost depends on the size of the project. The revenue obtained by the private firm is determined by the market value of the total credits issued, the credit ratio distributed to the private firm and the concession period that it can get free credits. Assume the concession period is predetermined and denote α as a parameter that transfers the capital cost of the project into unit period cost. The market value of the total credits issued is $\Pi = p\mathrm{K}$. Therefore, the profit that the private firm can obtain in a period is:

$$\Gamma(\kappa, y_l, v, q) = \gamma p \mathrm{K} - \alpha I(y_l) = \gamma \left(\sum_{w \in W} q_w B_w(q_w) - \sum_{a \in A, a \neq l} t_a(v_a) v_a - t_l(v_l, y_l) v_l \right) - \alpha I(y_l) \tag{7}$$

Furthermore, according to Yang and Wang (2011), the equilibrium credit price can be uniquely determined by the equation below:

$$p^* = \left(\sum_{w \in W} q_w^* B_w(q_w^*) - \sum_{a \in A} t_a(v_a^*) v_a^* \right) \Big/ \mathrm{K} \tag{8}$$

and as long as the credit price is positive, the credit distribution and charging scheme satisfies $\sum_{a \in A} \kappa_a v_a^* = \mathrm{K}$, where (v^*, q^*) are the equilibrium UE link flow and travel demand.

Yang and Wang (2011) have further shown that any tradable credits scheme contained in the following nonempty polyhedron can decentralize a given system optimum (SO) flow pattern (v^{so}, q^{so}) into UE:

$$\sum_{a \in A} (t_a(v_a^{so}) + \kappa_a) \delta_{a,r}^w \geqslant B_w(q_w^{so}), \quad r \in R_w, \ w \in W \tag{9}$$

$$\sum_{a \in A} (t_a(v_a^{so}) + \kappa_a) v_a^{so} = \sum_{w \in W} B_w(q_w^{so}) q_w^{so} \tag{10}$$

$$\sum_{a \in A} \kappa_a v_a^{so} = \mathrm{K}. \tag{11}$$

Under any tradable credits scheme contained in the above polyhedron, travelers' route choice behavior can be characterized by the following SO principle (Sheffi, 1985):

$$[\mathrm{SO}]\max_{(v,q)\in\Omega}\sum_{w\in W}\int_0^{q_w}B_w(\omega)d\omega-\sum_{a\in A}t_a(v_a)v_a \tag{12}$$

Therefore, if the tradable credits scheme satisfies the conditions (9)–(11), the corresponding lower level model in the BEC problem can be replaced by the above SO model.

Without loss of generality, in the following sections, we focus on two typical kinds of tradable credits scheme. In the first kind of tradable credits scheme, the credit charge of every link in the network changes after the new road is built. Furthermore, the feasible credit schemes adopted can always decentralize the SO flow pattern into UE. In the second kind of tradable credits scheme, only the credit charge of the new built link changes. The credit charges of other roads remain fixed. For convenience, we call the first kind of tradable credits scheme as the first-best tradable credits scheme and the second kind of tradable credits scheme as the second-best tradable credits scheme. Since only one link's credit charge needs to decide, the second-best scheme is very easy to carry out in practice while the first-best credit scheme can always provide SO flow pattern.

To facilitate the presentation of the essential ideas without loss of generality, we firstly present the following assumptions and these assumptions are used hereafter unless otherwise explicitly noted.

(A1) Travelers' route choices only depend on their generalized travel cost (inclusive travel time cost and credit charge cost). They do not change their route choices because of the variation of the credits distributed to them.
(A2) The private firm cannot control the credit price in the market by reducing the amount of credits it sells in the market.
(A3) Transaction cost and all the other administrative costs can be ignored. If these costs are sufficiently low compared to the costs of the credits, they are inconsequential in travelers' travel costs.
(A4) The link travel time function $t_a(v_a, y_a)$ is homogeneous of degree zero with respect to link flow and capacity.
(A5) There are constant returns to scale in the construction of the new road, namely, $I(y_l) = \lambda y_l$, where λ denote the cost for constructing one unit of capacity.
(A6) The reaction functions $v(\kappa, y_l)$ and $q(\kappa, y_l)$ defined by the lower level credit UE/SO problem are continuously differentiable with respect to the link capacity y_l and the link credit charge κ.

4. The BEC models under the first-best tradable credits scheme

4.1. Model formulation

As long as the credit scheme is contained in the polyhedron (9)–(11), the lower SO flow pattern and the upper objective function (social welfare or profit) do not depend on the specific credit scheme. Therefore, under the first-best tradable credits scheme, the social welfare maximization BEC problem (WM for short) can be written as:

$$[\mathrm{WM}]\quad \max_{y_l\geqslant 0,(v,q)\in\Omega}\sum_{w\in W}\int_0^{q_w}B_w(\omega)d\omega-\sum_{a\in A,a\neq l}t_a(v_a)v_a-t_l(v_l,y)v_l-\alpha I(y_l)$$

Note that the above social welfare maximization model is a single-level but not bi-level programming problem. This is because the upper level and the lower level have the same objective function in the social welfare maximization model under the first-best tradable credits scheme. However, if the objective is to maximize the profit of the private firm, the model is still a bi-level programming problem, which can be written as:

$$[\mathrm{PM}]\quad \max_{y_l\geqslant 0}\Gamma(y_l,v^{so},q^{so})=\gamma\left(\sum_{w\in W}q_w^{so}B_w(q_w^{so})-\sum_{a\in A}t_a(v_a^{so})v_a^{so}\right)-\alpha I(y_l)$$

where (v^{so}, q^{so}) can be obtained by solving the lower credit SO problem (10) with respect to a given vector of capacity y_l.

Existing literature has demonstrated that link service level provided in BOT is constant and only depends on its own travel time function and the unit construction cost (no matter the objective function is the social welfare or the profit of the private firm). Thus, it is significant to investigate if the same property still holds in BEC; if not, what factors determine the link service level. Furthermore, it has been proved that under assumptions A4-A5, self-financing holds in general network in BOT if all the roads of the network are tolled and their capacities are chosen in a socially optimal way (Yang and Meng, 2002). In BEC, the private firm receives its profit from all the travelers in the network. Although its profit depends on its credit distribution ratio and the total market value of the credits distributed. It is interesting to investigate if the total value of the credits charged on the new link can offset its own total construction cost and if the private firm can always get nonnegative profit.

4.2. Properties of the social welfare maximization BEC model

Since $y_l = 0$ implies that link l does not exist, we only consider the strictly positive capacity $y_l > 0$. Then the corresponding first-order condition of the social welfare maximization BEC problem is:

$$\frac{\partial Z}{\partial y_l^{so}} = \sum_{w \in W} B_w \frac{\partial q_w^{so}}{\partial y_l^{so}} - \sum_{a \in A}\left(t_a + \frac{\partial t_a(v_a^{so})}{\partial v_a^{so}} v_a^{so}\right)\frac{\partial v_a^{so}}{\partial y_l^{so}} - \frac{\partial t_l(y_l^{so})}{\partial y_l^{so}} v_l^{so} - \alpha\lambda = 0 \tag{13}$$

Under the first-best tradable credits scheme, the underlying flow pattern is always SO. For simplicity and consistency with the standard elastic-demand network equilibrium models in the literature, we will only discuss the case that $q_w < \hat{q}_w$ for all $w \in W$. Thus, at equilibrium, the following condition of the SO flow pattern can be obtained (Appendix A):

$$\sum_{a \in A}\left(t_a(v_a^{so}) + v_a^{so}\frac{\partial t_a}{\partial v_a^{so}}\right)\frac{\partial v_a^{so}}{\partial y_l^{so}} = \sum_{w \in W} B_w(q_w^{so})\frac{\partial q_w^{so}}{\partial y_l^{so}}. \tag{14}$$

According to Assumption 4, the link travel time only depends on the v/c ratio of the link. It leads to the following equation:

$$\frac{dt_l(v_l/y_l)}{d(v_l/y_l)} = y_l\frac{\partial t_l(v_l, y_l)}{\partial v_l} = -\frac{(y_l)^2}{v_l}\frac{\partial t_l(v_l, y_l)}{\partial y_l}. \tag{15}$$

Denote

$$g(v_l/y_l) = (v_l/y_l)^2\frac{dt_l(v_l/y_l)}{d(v_l/y_l)} \tag{16}$$

as a monotonically increasing function of v/c ratio. Combining Eqs. (13)–(15) yields:

$$g(v_l^{so}/y_l^{so}) = \alpha\lambda \tag{17}$$

Therefore, the new link's v/c ratio is constant and only depends on its own link travel time function and the unit construction cost. In other words, under the first-best tradable credits scheme, the social welfare maximization BEC model can provide constant link service level, which equals that offered in BOT (see Wu et al., 2011 for the link service level in BOT).

Under the first-best tradable credits scheme, the credit charge of every link in the network and the capacity of the new link are all socially optimal. From Eq. (17), the construction cost of the new link must satisfy:

$$\alpha\lambda y_l^{so} = -\frac{\partial t_l(y_l^{so})}{\partial y_l^{so}} v_l^{so} y_l^{so}. \tag{18}$$

Under SO flow pattern, the equilibrium credit price is equal to 1 (Yang and Wang, 2011). Combining Eq. (15), if the optimal credit charge of the new link satisfies the following condition, the total market value of the credits charged on the new link is just equal to its construction cost:

$$\kappa_l^{so} = \frac{\partial t_l(v_l^{so})}{\partial v_l^{so}} v_l^{so} = -\frac{\partial t_l(y_l^{so})}{\partial y_l^{so}} y_l^{so}. \tag{19}$$

Obviously, there exists at least one first-best credit charging scheme ($\kappa_a = \frac{\partial t_a(v_a^{so})}{\partial v_a^{so}} v_a^{so}, a \in A$) contained in the polyhedron (9)–(11) satisfying Eq. (19). Combining the property of the link service level, we can get:

Proposition 1. *Under the first-best tradable credits scheme, the link service level provided by the social welfare maximization BEC model is constant and equals that provided in BOT. Moreover, if the credit charge of the new link is equal to the marginal travel time of its flow, the total market value of the credits charged on the new link can just offset its construction cost.*

From Eq. (7), the profit of the private firm depends on the credit ratio γ. If γ is endogenous and satisfies $\gamma = \frac{\kappa_l^{so} v_l^{so}}{K}$, the profit of the private firm can be written as:

$$\Gamma = \kappa_l^{so} v_l^{so} - \alpha I(y_l^{so}). \tag{20}$$

According to Proposition 1, the total market value of the credits charged on the new link can just offset its construction cost if its credit charge is equal to the marginal travel time of its flow. Therefore, under the first-best tradable credits scheme, the social welfare maximization BEC model can provide constant link service level and the profit of the private firm is zero (positive) if the credit charge of the new link is equal to (larger than) the marginal travel time of its flow. On the other hand, if the credit ratio γ is exogenous and constant, we can always obtain a nonnegative profit by adjusting γ.[1]

[1] Hereafter we assume that the total market value of the credits distributed in the network is larger than the unit period construction cost of the new link.

4.3. Properties of the profit maximization BEC model

In the profit maximization BEC model, the profit is certainly nonnegative. This is obvious if the credit ratio γ is exogenous and constant as we can always adjusting γ. If γ is endogenous and the credits distributed to the private firm equal the total credits charged on the new link, the profit is also nonnegative. This is because the profit in the profit maximization must be no less than the zero profit in the social welfare maximization. Thus, in the profit maximization, the question is only the property of the link service level. We first consider the case that the credit ratio to the private firm is exogenous and constant. The first-order condition of the profit maximization problem with exogenous and constant credit distribution ratio is:

$$\gamma\left(\sum_{w\in W}\left(B_w + q_w^{so}\frac{\partial B_w}{\partial q_w}\right)\frac{\partial q_w^{so}}{\partial y_l^{so}} - \sum_{a\in A}\left(t_a + \frac{\partial t_a(v_a^{so})}{\partial v_a^{so}}v_a^{so}\right)\frac{\partial v_a^{so}}{\partial y_l^{so}} - \frac{\partial t_l(y_l^{so})}{\partial y_l^{so}}v_l^{so}\right) - \alpha\lambda = 0 \tag{21}$$

Substituting Eqs. (14)–(16) into the above condition yields:

$$g(v_l^{so}/y_l^{so}) = \frac{\alpha\lambda}{\gamma} - \sum_{w\in W} q_w^{so}\frac{\partial B_w}{\partial q_w^{so}}\frac{\partial q_w^{so}}{\partial y_l^{so}} = \frac{\alpha\lambda}{\gamma} - \sum_{w\in W} q_w^{so}\frac{\partial B_w}{\partial y_l^{so}}. \tag{22}$$

Given the capacity of the new link, the constant credit distribution ratio does not affect the SO flow pattern. Therefore, Eq. (22) means the v/c ratio decreases with the credit distribution ratio and the total marginal travel cost (inclusive of travel time cost and credit charge cost) of the new link's capacity. Therefore, we have the following proposition:

Proposition 2. *Under the first-best tradable credits scheme, if the credit ratio to the private firm is exogenous and constant, the link service level provided by the profit maximization BEC model is not constant. It depends on the credit ratio and the marginal travel cost (inclusive of travel time cost and credit charge cost) of the new link's capacity.*

Now we consider the case that the credit ratio is endogenous and the credits distributed to the private firm equals the total credits charged on the new link. Given $y_l^{so} > 0$, the link flow v_l^{so} and the construction cost $I(y_l^{so})$ are fixed. From Eq. (20), the profit of the private firm only depends on the credit charge of the new link. Denote $\widehat{f}_{r,w}, r \in R_w, w \in W$ as the route flow that uses the new link. $\widehat{v}_a, a \in A$ and $\widehat{q}_w, w \in W$ are the corresponding link flow and travel demand, respectively. We can get the maximal credit charge of the new link and the condition of the link v/c ratio (see Appendix B):

$$\kappa_l = \frac{1}{v_l^{so}}\left(\sum_{w\in W} B_w(q_w^{so})\widehat{q}_w^{so} - \sum_{a\in A} t_a(v_a^{so})\widehat{v}_a^{so}\right) \tag{23}$$

$$g(v_l^{so}/y_l^{so}) = \alpha\lambda - \sum_{w\in W}\frac{\partial B_w}{\partial y_l^{so}}\widehat{q}_w^{so} - \sum_{a\in A}\eta_a\widehat{v}_a^{so}\frac{\partial t_a(v_a^{so})}{\partial y_l^{so}}. \tag{24}$$

where $\eta_a = \left(\frac{\partial \widehat{v}_a^{so}}{\partial v_a^{so}}\frac{v_a^{so}}{\widehat{v}_a^{so}} - 1\right), a \in A$. $\frac{\partial \widehat{v}_a^{so}}{\partial v_a^{so}}\frac{v_a^{so}}{\widehat{v}_a^{so}}$ denotes for any infinitesimal change of the link flow, how much the link flow using the new link changes. Therefore, η_a means the additional change rate of the new link users on link $a \in A$. Obviously, the service level of the new link is not constant either in this case. Besides the construction cost and the new link's travel time function, it is also affected by other factors related to the new link users, such as the marginal travel time and the marginal travel cost (inclusive of travel time cost and credit charge cost) of the new link's capacity.

Proposition 3. *Under the first-best tradable credits scheme, if the credits distributed to the private firm equal the total amount of credits charged on the new link, the link service level offered by the profit maximization BEC model is not constant but depends on multiple factors related to the new link users.*

5. The BEC models under the second-best tradable credits scheme

5.1. Model formulation

Under the second-best tradable credits scheme, only the credit charge of the new link changes, the credit charges of all the other links are fixed. Therefore, only the credit charge and the capacity of the new link are variables and the underlying flow pattern is UE. Thus, the social welfare maximization BEC problem (W for short) can be built as:

$$[\mathrm{W}] \quad \max_{\kappa_l \geqslant 0, y_l \geqslant 0} Z(\kappa_l, y_l) = \sum_{w\in W}\int_0^{q_w^*} B_w(\omega)d\omega - \sum_{a\in A, a\neq l} t_a(v_a^*)v_a^* - t_l(v_l^*, y_l)v_l^* - \alpha I(y_l)$$

The profit maximization BEC problem (P for short) can be formulated as follow:

$$[\mathrm{P}] \quad \max_{\kappa_l \geqslant 0, y_l \geqslant 0} \Gamma(\kappa_l, y_l) = \gamma\left(\sum_{w\in W} q_w^* B_w(q_w^*) - \sum_{a\in A, a\neq l} t_a(v_a^*)v_a^* - t_l(v_l^*, y_l)v_l^*\right) - \alpha I(y_l)$$

where (v^*, q^*) can be obtained by solving the lower credit UE problem in Eqs. (4) and (5) with respect to a given vector of capacity and credit charge of the new link (κ_l, y_l).

5.2. Properties of the social welfare maximization BEC model

Because $y_l = 0$ implies that link l is not built, we only consider the strictly positive capacity $y_l > 0$. As discussed above, the construction cost function $I(y_l)$ is only the function of the link capacity y_l and is not affected by the credit charge κ_l. Thus, the corresponding first-order conditions of the social welfare maximization model [W] are:

$$\sum_{w\in W} B_w(q_w^*)\frac{\partial q_w^*}{\partial y_l^*} - \sum_{a\in A}\left(t_a(v_a^*) + \frac{\partial t_a(v_a^*)}{\partial v_a^*}v_a^*\right)\frac{\partial v_a^*}{\partial y_l^*} - \frac{\partial t_l(y_l^*)}{\partial y_l^*}v_l^* - \alpha\lambda = 0 \tag{25}$$

$$\sum_{w\in W} B_w(q_w^*)\frac{\partial q_w^*}{\partial \kappa_l^*} - \sum_{a\in A}\left(t_a(v_a^*)\frac{\partial v_a^*}{\partial \kappa_l^*} + \frac{\partial t_a(v_a^*)}{\partial \kappa_l^*}v_a^*\right) = 0. \tag{26}$$

We also only analyze the case that $q_w < \bar{q}_w$ for all $w \in W$. Furthermore, if the equilibrium credit price is equal to zero, the credit scheme is ineffective. Therefore, we only consider the case $p > 0$. Thus at equilibrium, we can get the following conditions about the UE flow pattern (Appendix C):

$$\sum_{a\in A} t_a(v_a^*)\frac{\partial v_a^*}{\partial y_l^*} = \sum_{w\in W} B_w(q_w^*)\frac{\partial q_w^*}{\partial y_l^*} \tag{27}$$

$$\sum_{a\in A} t_a(v_a^*)\frac{\partial v_a^*}{\partial \kappa_l^*} - \sum_{w\in W} B_w(q_w^*)\frac{\partial q_w^*}{\partial \kappa_l^*} = p^* v_l^*. \tag{28}$$

Substituting Eq. (27) into Eq. (25), we have:

$$\sum_{a\in A}\frac{\partial t_a(v_a^*)}{\partial v_a^*}\frac{\partial v_a^*}{\partial y_l^*}v_a^* + \frac{\partial t_l(v_l^*, y_l)}{\partial y_l^*}v_l^* = -\alpha\lambda. \tag{29}$$

Combining Eqs. (15) and (16) with Eq. (29) yields:

$$g(v_l^*/y_l^*) = \alpha\lambda + \sum_{a\in A} v_a^*\frac{\partial t_a(v_a^*)}{\partial y_l^*} \tag{30}$$

where $v_a^*\frac{\partial t_a(v_a^*)}{\partial y_l^*}, a \in A$ represent the marginal travel time of the new link's capacity. Thus, when the social welfare is maximized, the v/c ratio is not constant but depends on the total marginal travel time of the new link's capacity.

On the other hand, from Eq. (29), the construction cost of the new link satisfies:

$$\alpha\lambda y_l^* = \left(\sum_{a\neq l, \in A}\frac{\partial t_a(v_a^*)}{\partial y_l^*}v_a^* + \frac{\partial t_l(v_l^*, y_l)}{\partial y_l^*}v_l^*\right)y_l^* = y_l^*\sum_{a\in A}\frac{\partial t_a}{\partial y_l^*}v_a^*. \tag{31}$$

The total market value of the credits charged on the new link is $S = p^*\kappa_l^* v_l^*$. From Eqs. (26) and (28), it can be further written as:

$$S = \left(\sum_{a\in A} t_a\frac{\partial v_a^*}{\partial \kappa_l^*} - \sum_{w\in W} B_w\frac{\partial q_w^*}{\partial \kappa_l^*}\right)\kappa_l^* = \kappa_l^*\sum_{a\in A}\frac{\partial t_a}{\partial \kappa_l^*}v_a^*. \tag{32}$$

Therefore, if the following condition is satisfied, the total market value of the credits charged on the new link is just equal to its construction cost:

$$\kappa_l^*/y_l^* = \sum_{a\in A}\frac{\partial t_a}{\partial y_l^*}v_a^* \Bigg/ \sum_{a\in A}\frac{\partial t_a}{\partial \kappa_l^*}v_a^*. \tag{33}$$

The above condition means, if the ratio of the new link's optimal credit charge and its optimal capacity equals the reciprocal ratio of their total marginal travel times, the total market value of the credits charged on the new link can just offset its construction cost. Combining the above analyses yields the following proposition:

Proposition 4. *Under the second-best tradable credits scheme, the link service level provided by the social welfare maximization BEC model decreases with the total marginal travel time of the new link's capacity. Moreover, if the ratio of the new link's optimal credit charge and its optimal capacity equals the reciprocal ratio of their total marginal travel times, the total market value of the credits charged on the new link can just offset its construction cost.*

Under the second-best tradable credits scheme, the profit of the private firm can be nonnegative if the condition in Proposition 4 is satisfied and the credits distributed to the private firm equal the total credits charged on the new link. On the other hand, if the credit ratio γ is exogenous and constant, we can always obtain a nonnegative profit by adjusting γ.

5.3. Properties of the profit maximization BEC model

Like that under the first-best profit maximization BEC model, the profit here is nonnegative no matter γ is exogenous or endogenous. Thus, only the link service level of the new link is needed to analyze. We also first consider the case that γ is exogenous and constant. The first-order condition of the profit maximization program [P] with respect to the capacity of the new link is:

$$\gamma\left(\sum_{w\in W}\left(B_w(q_w^*)+q_w^*\frac{\partial B_w(q_w^*)}{\partial q_w}\right)\frac{\partial q_w^*}{\partial y_l^*}-\sum_{a\in A}\left(t_a(v_a^*)+\frac{\partial t_a(v_a^*)}{\partial v_a^*}v_a^*\right)\frac{\partial v_a^*}{\partial y_l^*}-\frac{\partial t_l(y_l^*)}{\partial y_l^*}v_l^*\right)-\alpha\lambda=0 \tag{34}$$

Substituting Eq. (27) into the above condition, we can get:

$$\gamma\left(\sum_{w\in W}q_w^*\frac{\partial B_w(q_w^*)}{\partial y_l^*}-\sum_{a\in A}\frac{\partial t_a(v_a^*)}{\partial v_a^*}\frac{\partial v_a^*}{\partial y_l^*}v_a^*-\frac{\partial t_l(y_l^*)}{\partial y_l^*}v_l^*\right)-\alpha\lambda=0. \tag{35}$$

On the other hand, substituting (15) and (16) into (34) yields

$$g(v_l^*/y_l^*)=\frac{\alpha\lambda}{\gamma}+\sum_{a\in A}\frac{\partial t_a(v_a^*)}{\partial y_l^*}v_a^*-\sum_{w\in W}q_w^*\frac{\partial B_w(q_w^*)}{\partial y_l^*}. \tag{36}$$

Given the credit charge scheme and the capacity of the new link, the traffic flow pattern depends on the credit distribution ratio γ, the total marginal travel time and the total marginal travel cost (inclusive of travel time cost and credit charge cost) of the new link's capacity. Based on Eq. (36), we have the following proposition:

Proposition 5. *Under the second-best tradable credits scheme, the link service level offered by the profit maximization BEC model depends on the credit distribution ratio, the total marginal travel time and the total marginal travel cost* (*inclusive of travel time cost and credit charge cost*) *of the new link's capacity*.

On the other hand, if the credits distributed to the private firm equal the total amount of credits charged on the new link, the objective function of the profit maximization model can be written as:

$$\Gamma=\frac{\kappa_l^*v_l^*}{\mathrm{K}}p^*\mathrm{K}-\alpha\lambda y_l^*=p^*\kappa_l^*v_l^*-\alpha\lambda y_l^*. \tag{37}$$

From Eq. (28), at equilibrium, $p^*v_l^*=\sum_{a\in A}t_a(v_a^*)\frac{\partial v_a^*}{\partial\kappa_l^*}-\sum_{w\in W}B_w(q_w^*)\frac{\partial q_w^*}{\partial\kappa_l^*}$. Substituting it into the objective function (37) yields:

$$\Gamma=\kappa_l^*\left(\sum_{a\in A}t_a\frac{\partial v_a^*}{\partial\kappa_l^*}-\sum_{w\in W}B_w\frac{\partial q_w^*}{\partial\kappa_l^*}\right)-\alpha\lambda y_l^*. \tag{38}$$

Taking the derivative of the above equation with respect to the capacity y_l, we have:

$$\kappa_l^*\sum_{a\in A}\left(\frac{\partial t_a}{\partial v_a^*}\frac{\partial v_a^*}{\partial y_l^*}\frac{\partial v_a^*}{\partial\kappa_l^*}+t_a\frac{\partial^2v_a^*}{\partial\kappa_l^*\partial y_l^*}\right)+\kappa_l^*\frac{\partial t_l}{\partial y_l^*}\frac{\partial v_l^*}{\partial\kappa_l^*}-\kappa_l^*\sum_{w\in W}\left(\frac{\partial B_w}{\partial q_w^*}\frac{\partial q_w^*}{\partial y_l^*}\frac{\partial q_w^*}{\partial\kappa_l^*}+B_w\frac{\partial^2q_w^*}{\partial\kappa_l^*\partial y_l^*}\right)-\alpha\lambda=0. \tag{39}$$

From Eq. (27), $\sum_{a\in A}t_a\frac{\partial v_a^*}{\partial y_l}=\sum_{w\in W}B_w\frac{\partial q_w^*}{\partial y_l}$. Take its derivative with respect to the credit charge κ_l:

$$\sum_{a\in A}\left(\frac{\partial t_a}{\partial v_a^*}\frac{\partial v_a^*}{\partial y_l^*}\frac{\partial v_a^*}{\partial\kappa_l^*}+t_a\frac{\partial^2v_a^*}{\partial\kappa_l^*\partial y_l^*}\right)=\sum_{w\in W}\left(\frac{\partial B_w}{\partial q_w^*}\frac{\partial q_w^*}{\partial y_l^*}\frac{\partial q_w^*}{\partial\kappa_l^*}+B_w\frac{\partial^2q_w^*}{\partial\kappa_l^*\partial y_l^*}\right). \tag{40}$$

Substituting Eq. (40) into Eq. (39), we have

$$\kappa_l^*\frac{\partial t_l}{\partial y_l^*}\frac{\partial v_l^*}{\partial\kappa_l^*}-\alpha\lambda=\frac{\kappa_l^*}{v_l^*}\frac{\partial v_l^*}{\partial\kappa_l^*}v_l^*\frac{\partial t_l}{\partial y_l^*}-\alpha\lambda=0. \tag{41}$$

Combining Eqs. (15) and (16) yields

$$g(v_l^*/y_l^*) = \alpha\lambda \Big/ \left(-\frac{\kappa_l^*}{v_l^*}\frac{\partial v_l^*}{\partial \kappa_l^*} \right) \tag{42}$$

where $\frac{\kappa_l^*}{v_l^*}\frac{\partial v_l^*}{\partial \kappa_l^*}$ represents the new link's credit charge elasticity in its flow. Therefore, the link service level of the new link depends on the new link's credit charge elasticity in its link flow (the elasticity of the new link's flow with respect to its credit charge). If it is smaller than -1, the link service level is higher than that in BOT; otherwise the link service level is equal or lower than that in BOT. Thus, we have the following proposition:

Proposition 6. *Under the second-best tradable credits scheme, if the credits distributed to the private firm equal the total credits charged on the new link, the link service level offered by profit maximization BEC model depends on the new link's credit charge elasticity in its flow.*

6. Discussion

In this paper, we propose and investigate a new private provision of public road: build-equity-credit, where the tradable credits scheme is employed to substitute the traditional road toll to get the profit for the private firm and further to manage the mobility. The properties of several different BEC models are investigated. The results are summarized in Table 1.[2]

(1) Tradable credits scheme vs. road tolls

Compared to the road tolls, the tradable credits scheme is considered to be more equitable and acceptable and no large financial transfer from travelers to the government exists. Moreover, the credits distribution scheme is a policy instrument for redistribution. The construction and maintenance costs borne by every traveler is trivial because of large number of users in the network. Furthermore, the tradable credits scheme can be designed to achieve any desired traffic flow pattern. After the concession period is over, the tradable credits scheme can continue to avoid severe congestion and explosion of the travel demand and the government can keep some credits to sell on the market so that it can get the fund for the management and maintenance of the road network afterwards.

Under the first-best tradable credits scheme, the underlying flow pattern is always SO. The corresponding tradable credits scheme is the same with the link toll scheme that can sustain the SO flow pattern (Yang and Wang, 2011). Therefore, the social welfare maximization BEC model plays the same role with the socially optimum BOT model (i.e., the SO model in Wu et al., 2011). Under the second-best tradable credits scheme, if the credits distributed to the private firm equal the amount of credits collected on the new link, the proposed model [P] is similar to the profit maximization model in BOT (e.g., the [PM] model in Wang et al., 2013). However, because the credit price affects travelers' travel cost, these two models are not equivalent.

(2) Link service level provided by the social welfare maximization model

The link service levels offered by the social welfare maximization BEC model and the socially optimum BOT model are equal, constant and only depend on the travel time function of the new link and the unit construction cost. However, under the second-best tradable credits scheme, the new add-on link changes every link's total marginal travel time through their link flows and further affects the total social welfare. Therefore, the link service level of the new link also depends on the total marginal travel time of the new link's capacity. As shown in Eq. (30), the link service level is higher than that in BOT if the total marginal travel time of the new link's capacity is negative; otherwise, the link service level is equal or lower. Generally speaking, the total marginal travel time of the new link's capacity is negative. Therefore, under the second-best tradable credits scheme, the link service level offered by the social welfare maximization model is generally higher than that in BOT.

(3) Link service level provided by the profit maximization model

Under the first-best tradable credits scheme, the link service level of the profit maximization model varies with the credit ratio and the marginal travel cost (inclusive the travel time cost and the credit charge cost) of the new link's capacity. The higher the credit distribution ratio to the private firm is, the higher the link service level is. Under the second-best tradable credits scheme, the link service level of the profit maximization model is also affected by these two factors. In addition, it is also affected by the total marginal travel time of the new link's capacity. Because the total marginal travel time of the new link's capacity is generally negative, the link service level under SO flow pattern is generally lower than that under UE flow pattern.

[2] For clarity and simplicity, hereafter we use variable credit distribution ratio to the private firm represents the special case that the amount of credits distributed to the private firm equals the total amount of credits charged on the new link, and constant credit distribution ratio to the private firm represents the general case.

Table 1
Properties of different BEC models.

		First-best Tradable Credits Scheme (BEC)			Second-best Tradable Credits Scheme (BEC)			Road tolls (BOT)	
Objective function		Social welfare	Profit		Social welfare	Profit		Social optimal	Profit
Credit ratio to the private firm		Constant/variable	Constant	Variable	Constant/variable	Constant	Variable	–	–
Variable	The capacity of the new link	√	√	√	√	√	√	√	√
	The credit charge of the new link				√	√	√		
	The toll of the new link							√	√
	The credit scheme			√					
Traffic flow pattern		SO	SO	SO	UE	UE	UE	SO	UE
Link service level of the new link		Constant	Variable	Variable	Variable	Variable	Variable	Constant	Constant
Impact factors of the link service level	Construction cost	√	√	√	√	√	√	√	√
	Travel time function	√	√	√	√	√	√	√	√
	Credit ratio		√			√			
	The total marginal travel time of the new link's capacity			√[a]	√	√			
	The total marginal travel cost of the new link's capacity		√	√[a]		√			×
	The new link's credit charge elasticity in its flow						√		
Self-financing of the new link[b]		Conditionally	Nonnegative	Nonnegative	Conditionally	Nonnegative	Nonnegative	Conditionally	Nonnegative

[a] Only related to the new link users.
[b] Here self-financing means the total market value of the credits charged on the new link can offset its construction cost.

As a special case, if the credits distributed to the private firm equal the total amount of credits charged on the new link, the link service level provided by the profit maximization model is affected by different factors. Under the first-best tradable credits scheme, the link service level is affected by the total marginal travel time of the additional new link users and the total marginal travel cost (inclusive travel time cost and credit charge cost) of the new link users with respect to the new link's capacity. Under the second-best tradable credits scheme, the link service level depends on the new link's credit charge elasticity in its flow. If the new link's flow is elastic (smaller than -1) with respect to its credit charge, the link service level is higher than that in BOT; otherwise, the link service level is equal or lower than that in BOT.

(4) The total value of the credits charged on the new link and the profit of the private firm

Under the first-best tradable credits scheme, if the credit charge of the new link is equal to the marginal travel time of its flow, the total market value of the credits charged on the new link can just offset its construction cost. Under the second-best tradable credits scheme, if the ratio of the new link's optimal credit charge and its optimal capacity equals the reciprocal ratio of their total marginal travel times, the total market value of the credits charged on the new link can just offset its construction cost. Under these conditions, if the credits distributed to the private firm equal the total credits charged on the new link, the profit of the private firm is zero. If the credit distribution ratio to the private firm is constant, a nonnegative profit can always be obtained by adjusting the credit distribution ratio.

On the other hand, the nonnegative profit of the private firm can always be ensured in the profit maximization BEC model. If the credit distribution ratio to the private firm is endogenous and the credits distributed to the private firm equal the total credits charged on the new link, maximizing the profit must produce a nonnegative profit and the total market value of the credits charged on the new link can offset its construction cost. On the other hand, if the credit distribution ratio to the private firm is exogenous and constant, we can always get a nonnegative profit by adjusting the credit distribution ratio.

(5) Policy implications

The social welfare maximization BEC model under the first-best tradable credits scheme can offer the largest social welfare and constant link service level, which is equal to that in BOT. However, if the government wants to achieve a higher link service level and can ensure a negative total marginal travel time of the new link's capacity, it can choose the second-best tradable credits scheme to manage mobility. If the profit of the private firm is the main objective the government considers, it can raise the link service level by setting a higher credit distribution ratio. However, higher credit distribution ratio to the private firm will reduce the benefit of the travelers as their share of credits decreases. In addition, the link service level under the second-best tradable credits scheme is higher than that under the first-best tradable credits scheme. If the credits distributed to the private firm equal the total credits collected on the new link, the new link's credit charge elasticity in its flow under the second-best tradable credits scheme must be considered. If the new link's flow is very elastic with respect to its credit charge, the resulted link service level is higher than the constant link service level; otherwise, it is equal or lower. The link service under the first-best tradable credits scheme depends on multiple factors related to the new link travelers.

7. Numerical example

In the following, we use a simple example to illustrate the properties of the several different BEC models. Consider the network shown in Fig. 4 that contains one OD pair with travel demand function $q = 100\exp(-0.1\mu)$, where μ is the minimal generalized travel cost. For each link, the number outside the bracket is the link number and the number inside the bracket is its credit charge under the second-best tradable credits scheme. Link 6 is the new link with its capacity as y. Assume $\alpha = 1$ and $I = 0.5y$. The link travel time functions are given by

$$t_1 = 1 + v_1/40, \quad t_2 = 2 + v_2/20, \quad t_3 = 0.5 + v_3/10, \quad t_4 = 2 + v_4/20, \quad t_5 = 1 + v_5/40, \quad t_6 = 3 + v_6/y.$$

For the profit maximization BEC model, we consider two cases: a general one with a constant credit distribution ratio to the private firm and a special case, where the total credits distributed to the private firm is equal to the total amount of credits charged on the new link (which we call as variable credit distribution ratio for convenience). Assume that the constant credit ratio distributed to the private firm is $\gamma = 0.2$ and the total amount of credits distributed under the second-best tradable credits scheme is K = 280. Table 2 gives the new link's v/c ratio, the profit of the private firm and the total social welfare under different scenarios. In the following, we analyze the properties of the different scenarios in BEC and compare them with the corresponding results in BOT.

As expected, the link service level in the BOT models only depends on the construction cost and the travel function of the new link. Under the first-best tradable credits scheme, the link service level provided by the social welfare maximization is equal to that in BOT. In addition, under the same objective, the resulted link service level is higher under the second-best tradable credits scheme (UE flow pattern) than that under the first-best tradable credits scheme (SO flow pattern) in BEC. Especially, the profit maximization model with variable credit ratio under UE flow pattern can offer a much higher link service level than all the other scenarios. As described in Table 1, the profit maximization provision of link service level depends

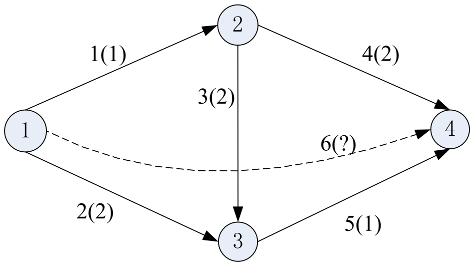

Fig. 4. A simple network.

Table 2
Results under different scenarios.

	First-best Tradable Credits Scheme			Second-best Tradable Credits Scheme			Road toll (BOT)[b]	
Objective	Social welfare	Profit		Social welfare	Profit		Social welfare	Profit
Credit ratio	–	Constant	Variable	–	Constant	Variable	–	–
v/c ratio	0.71	1.83	1.22	0.67	1.19	0.58	0.71	0.71
Profit	2.78/0[a]	18.87	19.03	−29.23/−15.03[a]	4.26	11.51	0	24.24
Welfare	658.13	608.32	642.40	657.12	625.84	643.60	658.13	645.42

[a] The former number denotes the profit of the private firm with constant credit distribution ratio and the latter denotes the profit of the private firm with variable credit distribution ratio.
[b] the tolls of the old links are first-best.

on the new link's credit charge elasticity in its flow. Therefore, the optimal solutions imply the new link's flow is very elastic with respect to its credit charge in this example.

In addition, no matter whether the underlying flow pattern is UE or SO, the link service level of the profit maximization model with constant credit distribution ratio is always the lowest among all the scenarios. This is because, compared to other scenarios, its provision of link service level is also affected by the credit distribution ratio to the private firm. According to Eqs. (22) and (36), the smaller the credit ratio is, the lower the link service level is. In this example, the credit ratio is 0.2, much smaller than 1, so the link service level provided by the general profit maximization model is lower than other scenarios.

It may be strange to notice that the social welfare is higher under UE flow pattern than that under SO flow pattern when the objective is profit maximization (both constant credit ratio and variable credit ratio). This is because the SO underlying flow pattern can only ensure a higher total social welfare when the capacity of the new link is given. When the capacity of the new link is variable, the social welfare under SO flow pattern may not be larger than that with UE flow pattern. In fact, only the social welfare maximization model under SO flow pattern can certainly provide the largest social welfare.

Table 2 also presents the profit of the private firm under different scenarios. In the social optimum BOT model, the new link is self-financing, therefore, the profit of the private firm is equal to 0. Under the first-best tradable credits scheme, if the credits distributed to the private firm equals the total credits charged on the new link, the profit of the private firm in the social welfare maximization BEC model is also 0. However, under the second-best tradable credits scheme, the profit in the social welfare maximization BEC model is negative, as the condition in Proposition 4 is not satisfied. The results also show that the profits under SO flow pattern is higher than that under UE flow pattern in BEC. Furthermore, in the profit maximization BEC model, the private firm can obtain a higher profit if the total credits distributed to it equal the amount of credits collected on the new link.

8. Conclusions and future research

In this paper, we investigate a new kind of private financing of the public road. The main contributions of this paper to the literature are as follows:

- A new private provision of public road: the build-equity-credit (BEC) scheme, is explored, which is a combination of private financing and mobility management. Instead of road tolls, the tradable credits scheme, which is thought to be a more equitable and acceptable traffic management instrument, is used to manage mobility and gain the profit of the private firm. After the private firm gets its expected profit, the tradable credits scheme will continue to prevent severe congestion and explosion of travel demand. The government can keep some credits to sell in the market to get fund for the road management and maintenance afterwards.

- A general bi-level programming problem is formulated to model the determination of the tradable credits scheme and the capacity of the new road. Two typical kinds of tradable credits schemes are proposed. The first-best tradable credits scheme can always decentralize the SO flow pattern into UE and represents the SO flow pattern case. The second-best tradable credits scheme represents the general case that the underlying flow pattern is UE. Several different models are formulated based on different objectives under these two kinds of tradable credits scheme.
- The properties of different BEC models are investigated. The link service level is generally not constant in BEC. Its influential factors are multiple. The unit construction cost and the travel time function of the new link are always the principal influential factors of the link service level. Other influential factors include the marginal travel times and the marginal travel cost (inclusive of travel time cost and credit charge cost) of the new link's capacity and the new link's credit charge elasticity in its flow. The total market value of the credits charged on the new link can offset its construction cots under some conditions and the profit of the private firm can be ensured to be nonnegative.

Further extensions can be carried out based on the proposed models and analyses. In this paper, we only consider the extreme cases that either the social welfare or the profit of the private firm is maximized. The more common case is that the objective function is the trade-off between the social welfare and the profit of the private firm. Therefore, the properties of the trade-off of these two extreme objectives need further investigation.

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Appendix A. Property of the SO link flow pattern

Given the capacity of the new link y_l, the following first-order conditions for the SO program hold:

$$\sum_{a\in A,a\neq l}\left(t_a(v_a^{so})+v_a\frac{\partial t_a(v_a^{so})}{\partial v_a^{so}}\right)\delta_{a,r}^w+\left(t_l+v_l^{so}\frac{\partial t_l(v_l^{so},y_l)}{\partial v_l^{so}}\right)\delta_{l,r}^w=B_w(q_w^{so}),\quad \text{if } f_{r,w}>0,\ r\in R_w,\ w\in W \tag{A1}$$

$$\sum_{a\in A,a\neq l}\left(t_a(v_a^{so})+v_a^{so}\frac{\partial t_a(v_a^{so})}{\partial v_a^{so}}\right)\delta_{a,r}^w+\left(t_l+v_l\frac{\partial t_l(v_l^{so},y_l)}{\partial v_l^{so}}\right)\delta_{l,r}^w\geqslant B_w(q_w^{so}),\quad \text{if } f_{r,w}=0,\ r\in R_w,\ w\in W. \tag{A2}$$

Summing Eq. (A1) over all $r\in R_w$ and $w\in W$ and using the definitions $\sum_{w\in W}\sum_{r\in R_w}f_{r,w}\delta_{a,r}^w=v_a$ and $\sum_{r\in R_w}f_{r,w}=q_w$, we have

$$\sum_{a\in A,a\neq l}\left(t_a(v_a^{so})+v_a^{so}\frac{\partial t_a(v_a^{so})}{\partial v_a^{so}}\right)v_a^{so}+\left(t_l(v_a^{so},y_l)+v_l^{so}\frac{\partial t_l(v_a^{so},y_l)}{\partial v_l^{so}}\right)v_l^{so}=\sum_{w\in W}q_w^{so}B_w(q_w^{so}),\quad r\in R_w,w\in W. \tag{A3}$$

Taking the derivative of the both sides of Eq. (A3) with respect to y_l yields

$$\sum_{a\in A}\left(t_a+3v_a^{so}\frac{\partial t_a}{\partial v_a^{so}}+(v_a^{so})^2\frac{\partial^2 t_a}{\partial(v_a^{so})^2}\right)\frac{\partial v_a^{so}}{\partial y_l}+(v_l^{so})^2\frac{\partial^2 t_l}{\partial v_a^{so}\partial y_l}+v_l^{so}\frac{\partial t_l}{\partial y_l}=\sum_{w\in W}\left(B_w+q_w^{so}\frac{\partial B_w}{\partial q_w^{so}}\right)\frac{\partial q_w^{so}}{\partial y_l} \tag{A4}$$

Taking the derivative of the both sides of Eq. (A1) with respect to y_l yields

$$\sum_{a\in A}\left(2\frac{\partial t_a}{\partial v_a^{so}}+v_a^{so}\frac{\partial^2 t_a}{\partial(v_a^{so})^2}\right)\frac{\partial v_a^{so}}{\partial y_l}\delta_{a,r}^w+\left(v_l^{so}\frac{\partial^2 t_l}{\partial v_l^{so}\partial y_l}+\frac{\partial t_l}{\partial y_l}\right)\delta_{l,r}^w=\frac{\partial B_w}{\partial q_w^{so}}\frac{\partial q_w^{so}}{\partial y_l},\quad r\in R_w,\ w\in W \tag{A5}$$

Multiplying both sides of Eq. (A5) with f_r^w and adding them up over all $r\in R_w, w\in W$ results in

$$\sum_{a\in A}\left(2\frac{\partial t_a}{\partial v_a^{so}}+v_a^{so}\frac{\partial^2 t_a}{\partial(v_a^{so})^2}\right)\frac{\partial v_a^{so}}{\partial y_l}v_a^{so}+\left(v_l^{so}\frac{\partial^2 t_l}{\partial v_l^{so}\partial y_l}+\frac{\partial t_l}{\partial y_l}\right)v_l^{so}=\sum_{w\in W}q_w^{so}\frac{\partial B_w}{\partial q_w^{so}}\frac{\partial q_w^{so}}{\partial y_l}. \tag{A6}$$

Combining Eqs. (A4) and (A6) yields Eq. (14).

Appendix B. Property of the profit maximization BEC model with endogenous credit ratio

Given $y_l^{so}\geqslant 0$, the link flow v_l^{so} and the construction cost $I(y_l^{so})$ are fixed, the profit of the private firm only depends on the credit charge of the new link. To obtain the maximal profit, the credit charge κ_l must be maximized. Denote $\widetilde{v}_a=v_a^{so}(y_l), a\in A$ and $\widetilde{q}_w=q_w^{so}(y_l), w\in W$, we formulate the following linear programming (LP) problem to get the maximal κ_l:

$$[\text{LP}]\quad \max_{\kappa_l} \kappa_l$$

subject to.

$$\sum_{a\in A}\kappa_a\delta_{a,r}^w \geqslant B_w(\tilde{q}_w) - \sum_{a\in A} t_a(\tilde{v}_a)\delta_{a,r}^w, \quad r\in R_w,\ w\in W$$

$$\sum_{a\in A}\kappa_a\tilde{v}_a = \sum_{w\in W} B_w(\tilde{q}_w)\tilde{q}_w - \sum_{a\in A} t_a(\tilde{v}_a)\tilde{v}_a$$

$$\kappa_a \geqslant 0,\ a\in A.$$

The dual formulation of the above LP problem is:

$$[\text{du}]\quad \min_{x,z} L = -\sum_{w\in W}\sum_{r\in R_w}\left(B_w(\tilde{q}_w) - \sum_{a\in A} t_a(\tilde{v}_a)\delta_{a,r}^w\right)x_{r,w} + \left(\sum_{w\in W} B_w(\tilde{q}_w)\tilde{q}_w - \sum_{a\in A} t_a(\tilde{v}_a)\tilde{v}_a\right)z$$

$$\sum_{w\in W}\sum_{r\in R_w} x_{r,w}\delta_{a,r}^w - z\tilde{v}_a \leqslant 0, a\in A, a\neq l$$

$$\sum_{w\in W}\sum_{r\in R_w} x_{r,w}\delta_{l,r}^w - z\tilde{v}_l \leqslant -1$$

$$x_{r,w} \geqslant 0,\quad r\in R_w,\ w\in W$$

where the dual variables $x = (x_{r,w}, r\in R_w, w\in W)$ and z are associated with the inequality constraints and the equality constraint in the primal LP problem, respectively, and z is unrestricted in sign. Let $\hat{f}_{r,w} = \begin{cases} \tilde{f}_{r,w}, & \text{if } \delta_{l,r}^w = 0 \\ 0, & \text{otherwise} \end{cases},\quad r\in R_w,\ w\in W$ denote the flow that does not use the new link and $z = \frac{1}{\tilde{v}_l}$. It can be proved that $(\hat{f}/v_l, z)$ is the optimal solution to the dual problem [DP] (see Appendix D). According to the duality theory, the primal LP problem and the dual problem have the same optimal value, therefore, the maximal credit charge κ_l equals to:

$$\kappa_l = -\frac{1}{\tilde{v}_l}\sum_{w\in W}\sum_{r\in R_w}\left(B_w(\tilde{q}_w) - \sum_{a\in A} t_a(\tilde{v}_a)\delta_{a,r}^w\right)\hat{f}_{r,w} + \frac{1}{\tilde{v}_l}\left(\sum_{w\in W} B_w(\tilde{q}_w)\tilde{q}_w - \sum_{a\in A} t_a(\tilde{v}_a)\tilde{v}_a\right) \tag{B1}$$

Denote $\hat{v}_a = \sum_{w\in W}\sum_{r\in R_w}\hat{f}_{r,w}\delta_{a,r}^w, a\in A$ and $\hat{q}_w = \sum_{r\in R_w}\hat{f}_{r,w}, w\in W$, the maximal credit charge κ_l can be rewrite as Eq. (23). Substituting the above equation into the profit function and we have:

$$[\text{PM}']\quad \max_{y_l}\Gamma_l^2 = \sum_{w\in W} B_w(q_w^{so})(q_w^{so} - \hat{q}_w^{so}) - \sum_{a\in A} t_a(v_a^{so})(v_a^{so} - \hat{v}_a^{so}) - \alpha I(y_l)$$

subject to

$$y_l \geqslant 0.$$

Using the similar method with that in Appendix A, we can get the below condition:

$$\sum_{a\in A}\left(t_a(v_a^{so}) + v_a^{so}\frac{\partial t_a}{\partial v_a^{so}}\right)\frac{\partial \hat{v}_a^{so}}{\partial y_l} = \sum_{w\in W} B_w(q_w^{so})\frac{\partial \hat{q}_w^{so}}{\partial y_l} \tag{B2}$$

We also only consider the case that $y_l > 0$, and combing Eqs. (14) and (B2), the first order KKT condition of [PM′] can be written as:

$$\frac{\partial \Gamma_l^2}{\partial y_l} = \sum_{w\in W}\frac{\partial B_w}{\partial y_l}(q_w^{so} - \hat{q}_w^{so}) - \sum_{a\in A}\frac{\partial t_a}{\partial v_a^{so}}\left(\frac{\partial \hat{v}_a}{\partial y_l}v_a^{so} - \frac{\partial v_a}{\partial y_l}\hat{v}_a^{so}\right) - \frac{\partial t_l}{\partial y_l}v_l^{so} - \alpha\lambda = 0. \tag{B3}$$

Substituting Eq. (15) into the above condition and we get:

$$g(v_l^{so}/y_l) = \alpha\lambda - \sum_{w\in W}\frac{\partial B_w}{\partial y_l}(q_w^{so} - \hat{q}_w^{so}) + \sum_{a\in A}\frac{\partial t_a}{\partial v_a^{so}}\left(\frac{\partial \hat{v}_a}{\partial y_l}v_a^{so} - \frac{\partial v_a}{\partial y_l}\hat{v}_a^{so}\right). \tag{B4}$$

Denote $\hat{q}_w^{so} = q_w^{so} - \hat{q}_w^{so}, w\in W,\ \hat{v}_a^{so} = v_a^{so} - \hat{v}_a^{so}, a\in A$, we have

$$g(v_l^{so}/y_l^{so}) = \alpha\lambda - \sum_{w\in W}\frac{\partial B_w}{\partial y_l^{so}}\hat{q}_w^{so} - \sum_{a\in A}\frac{\partial t_a(v_a^{so})}{\partial v_a^{so}}\left(\frac{\partial \hat{v}_a^{so}}{\partial y_l^{so}}v_a^{so} - \frac{\partial v_a^{so}}{\partial y_l^{so}}\hat{v}_a^{so}\right) \tag{B5}$$

which can be further written as Eq. (24).

Appendix C. Properties of the credit UE flow pattern

Given the capacity and the credit charge of the new link (κ_l, y_l), the first-order conditions of the lower credit UE problem (3)–(7) can be written as:

$$\sum_{a\in A, a\neq l}(t_a(v_a^*)+p^*\kappa_a)\delta_{a,r}^w+(t_l(v_l^*,y_l)+p^*\kappa_l)\delta_{l,r}^w=B_w(q_w^*),\quad \text{if } f_{r,w}>0,\ r\in R_w,\ w\in W \tag{C1}$$

$$\sum_{a\in A, a\neq l}(t_a(v_a^*)+p^*\kappa_a)\delta_{a,r}^w+(t_l(v_l^*,y_l)+p^*\kappa_l)\delta_{l,r}^w\geqslant B_w(q_w^*),\quad \text{if } f_{r,w}=0,\ r\in R_w,\ w\in W \tag{C2}$$

$$\sum_{a\in A}\kappa_a v_a^*=\mathrm{K},\quad p^*>0. \tag{C3}$$

Using the similar method in Appendix A, we can obtain the following condition:

$$\sum_{a\in A}\left(t_a(v_a^*)\frac{\partial v_a^*}{\partial y_l}+p^*\kappa_a\frac{\partial v_a^*}{\partial y_l}\right)=\sum_{w\in W}B_w(q_w^*)\frac{\partial q_w^*}{\partial y_l} \tag{C4}$$

$$\sum_{a\in A}\left(t_a\frac{\partial v_a^*}{\partial \kappa_l}+p^*\kappa_a\frac{\partial v_a^*}{\partial \kappa_l}\right)=\sum_{w\in W}B_w\frac{\partial q_w^*}{\partial \kappa_l}. \tag{C5}$$

As the total amount of credits K and the link credit charge $\kappa_a, a\neq l$ remains unaltered with respect to y_l and κ_l, therefore, from Eq. (C3), we have $\sum_{a\in A}\kappa_a\frac{\partial v_a^*}{\partial y_l}=0$ and $\sum_{a\in A}\kappa_a\frac{\partial v_a^*}{\partial \kappa_l}+v_l^*=0$. Substituting them into Eqs. (C4) and (C5) yields Eqs. (27) and (28).

Appendix D. $(\hat{f},\hat{z})$ is the optimal solution to the dual problem [du]

Proof. Firstly, we can confirm that $(\hat{f},\hat{z})$ is a feasible solution to the dual problem [du]. According to the first-order conditions of the linear programming problem, if $(\hat{f},\hat{z})$ is the optimal solution to [du], there must exist multiplier vectors $\sigma=(\sigma_a, a\in A)$ and $\varsigma=(\varsigma_{r,w}, r\in R_w, w\in W)$ such that

$$-(B_w(\hat{q}_w)-\underset{a\in A}{m}t_a(\hat{v}_a)\delta_{a,r}^w)+\sum_{a\in A}\sigma_a\delta_{a,r}^w-\sigma_{r,w}=0,\quad r\in R_w,\ w\in W \tag{D1}$$

$$\sigma_a\left(\sum_{w\in W}\sum_{r\in R_w}\hat{f}_{r,w}\delta_{a,r}^w-\hat{z}\hat{v}_a\right)=0,\quad \sum_{w\in W}\sum_{r\in R_w}\hat{f}_{r,w}\delta_{a,r}^w-\hat{z}\hat{v}_a\leqslant 0,\quad \sigma_a\geqslant 0,\ a\in A,\ a\neq l \tag{D2}$$

$$\sigma_l\left(\sum_{w\in W}\sum_{r\in R_w}\hat{f}_{r,w}\delta_{l,r}^w-\hat{z}\hat{v}_l+1\right)=0,\quad \sum_{w\in W}\sum_{r\in R_w}x_{r,w}\delta_{l,r}^w-\hat{z}\hat{v}_l\leqslant -1,\ \sigma_l\geqslant 0 \tag{D3}$$

$$\varsigma_{r,w}\hat{f}_{r,w}=0,\quad \hat{f}_{r,w}\geqslant 0,\ \varsigma_{r,w}\geqslant 0,\ r\in R_w,\ w\in W \tag{D4}$$

Let $\sigma_a=\hat{v}_a\frac{\partial t_a}{\partial v_a}, a\in A$ and $\varsigma_{r,w}=\begin{cases}\sum_{a\in A}\left(t_a(\hat{v}_a)+\hat{v}_a\frac{\partial t_a}{\partial v_a}\right)\delta_{a,r}^w-B_w(\hat{q}_w), & \text{if } \hat{f}_{r,w}=0\\ 0, \text{otherwise}\end{cases}$, which satisfy D1-D4. Therefore, $(\hat{f},\hat{z})$ is the optimal solution to [du]. This completes the proof. □

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