Information Sciences 373 (2016) 287–307

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Robust train regulation for metro lines with stochastic passenger arrival flow

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ARTICLE INFO

Article history:
Received 8 July 2015
Revised 4 August 2016
Accepted 5 September 2016
Available online 6 September 2016

Keywords:
Train regulation
Stochastic passenger flow
Discrete Markovian process
Robust state-feedback control

ABSTRACT

This paper investigates the robust train regulation problem for metro lines with a stochastic passenger arrival flow. The passenger arrival flow is assumed to be dependent on a discrete Markovian process. A constrained state-space model for the train traffic of a metro-line operation is developed from a system-theoretic standpoint. According to stochastic stability theory, we give a sufficient condition for the existence of state-feedback control as the train regulation strategy in terms of linear matrix inequalities, which ensures the stochastic stability of the train traffic of metro lines. By considering the uncertain disturbances to the train operation, a robust train regulation strategy that guarantees that the practical train timetable tracks the nominal timetable with a disturbance attenuation level is designed, and the total delays of the trains at each station are reduced. Moreover, a nonlinear optimization problem is formulated to determine the optimal robust train regulation strategy that ensures the minimization of the disturbance attenuation level. Numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

1.1. Motivation

Urban metro transportation systems are an attractive mode of transports for relieving the traffic pressure in modern large cities due to their inherent features of reliability, energy efficiency, sustainability and high capacity. It is well known that high-frequency metro lines are naturally unstable because any deviation with respect to the nominal schedule of a given train will be amplified with time, and consequently the operation of other trains will be disturbed [5,27]. On a high-frequency metro line, passengers arrive randomly at the stations, and the train delays increase at each station with the accumulation of passengers, leading to the instability of metro lines. In addition, the uncertain disturbances to the running time and dwell time of the train will also lead to instabilities of metro lines, such as system abnormality, inadequate driver/passenger action and so on [22]. By manipulating the running time and the dwell time of each train, train regulation attempts to recover train delays and prevent the instability of metro line operations. Moreover, because of their intrinsically stochastic characteristic and instability, the robustness of the train regulation system against the stochastic passenger flow fluctuations and uncertain disturbances is an important factor affecting the capacity utilization and the service quality for

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http://dx.doi.org/10.1016/j.ins.2016.09.019
0020-0255/

highly homogeneous metro lines. Therefore, a robust train regulation design is necessary to prevent such instabilities, both from the passenger and company viewpoints.

The discrete event model is a more suitable class of mathematical models for describing the dynamic performance of the train traffic of metro lines [27]. The corresponding variables are related to both trains and stations. The train traffic model based on discrete events can be regarded as a discrete-stage equation, which can well describe the dynamic train traffic flow of metro lines. Based on the discrete-stage equation, one can apply modern control theory to address the stability condition of the train traffic. In addition, in contrast to large-scale transportation systems mainly involving dynamic vehicle flows from a macroscopic perspective [21,29], the passenger arrival flow will affect the stability of high-frequency metro lines. Therefore, it is necessary to consider the effects of both vehicle flow (trains) and passenger flow for metro lines. Consider that the passenger arrival rate is stochastically changing at different stages of stations. Markovian processes, with a characteristic property that is sometimes stated as “the future depends on the past only through the present”, are good models for many stochastic systems, including certain queuing systems, inventory systems and reliable systems [15,23]. Similarly, as a special case of queuing systems, the passenger arrival rate at different stages of stations can be assumed to be dependent on a discrete Markovian process, and the probability transition matrix will be obtained using historical data. Due to effects of the stochastic passenger flow on the dwell time of a train, a train traffic model considering the dwell time of the train is therefore formulated using a stochastic discrete dynamic system with Markovian jumping parameters, and the stochastic stability will be derived using stochastic stability theory [11]. Moreover, we can denote the uncertain disturbance to the train delays as an unknown-but-bounded quantity with finite energy [27]. Then, the so-called robust control theory can be applied to guarantee that the train traffic model has a smaller prescribed disturbance attenuation level with respect to the uncertain disturbances [10,31]. This motivates the idea to design a robust train regulation strategy for metro lines with stochastic passenger arrival flows and uncertain disturbances within the framework of robust control theory.

1.2. Some related literature and contributions

In train regulation, the buffer times or supplements in the timetable are usually designed to absorb the train delays resulting from disturbances [1,28]. However, the buffer time allocation is static and cannot be used dynamically and flexibly from a system-wide point of view. This may reduce system capacity utilization for the possible redundant buffer time. In particular, automatic train regulation (ATR) is used to recover the schedule/headway deviations resulting from disturbances by dynamically adjusting the running time and dwell time of each train in real time, thereby reducing the potential redundant buffer time and improving the system capacity utilizations. Recently, a number of automatic train regulation methods were proposed for metro lines [6,7,13,16,32]. Van Breusegem et al. [27] proposed a complete discrete-event traffic model for metro lines, in which the state feedback control algorithm was designed by solving an optimization unconstrained quadratic problem, which ensures the system stability and the minimization of a given performance index. Followed by the discrete-event traffic model, Chang and Chung [6] applied a genetic algorithm to efficiently solve the train rescheduling problem. By quadratic programming, Fernandez et al. [12] addressed a predictive traffic regulation model for metro loop lines to optimize a cost function along a time horizon, and the proposed quadratic programming model can be solved efficiently in real time. A new methodology for the computation of the optimal train schedules in metro lines was proposed by Assis and Milani [2] with a linear-programming-based model predictive control, which generates the optimal schedule for a whole day operation. From the passenger perspective, Goodman and Murata [14] considered a constrained nonlinear programming for metro traffic regulation. Dorfman and Medanic [9] developed a local feedback-based travel advance strategy for train advances along lines of the railway, and the proposed approach can quickly handle the perturbations in the schedule. By using dual heuristic dynamic programming, Lin and Sheu [22] proposed an automatic train regulation, and a near-optimal regulation was obtained more rapidly and accurately. A cooperative scheduling approach was developed to optimize the timetable by Yang et al. [30], in which the recovery energy generated by braking train can be directly used by accelerating train, which achieved the energy-saving of the subway systems.

Clearly, many efficient train regulation techniques have been proposed to optimize the performance index for metro lines. However, in most studies, the passenger arrival flow at the stations is assumed to be a constant or pre-known variable [2,22,27]. In practice, the passenger arrival flow is dynamically and stochastically changing at the stations, especially during peak hours. It is reasonable to apply a stochastic process to describe the dynamic changing of the passenger arrival flow. Moreover, the uncertain disturbances will lead to an instability of high-frequency metro lines. Therefore, the robustness of train regulation against uncertain disturbances is an important problem for improving the capacity utilization and service quality of metro lines. To address this problem, the robustness of train regulation problems has received substantial attention by researchers, By considering a service disruption on a single-track rail line, Meng and Zhou [24] proposed a robust disruption handling method to minimize the expected additional delay under different forecasted operational conditions based on a stochastic programming method. Shafia et al. [26] derived a new robust train-timetabling problem in a single-track railway line, and presented a branch-and-bound algorithm along with a new heuristic beam search algorithm to solve the model for large-scale problems, which can effectively find a near-optimum solution in a rational amount of time. The existing literature on the robustness of the train regulation problem against uncertain disturbances mainly addresses small train delays using a buffer time. However, for larger train delays, few studies can be found to address the robust train regulation problem for metro line systems with uncertain disturbances. For this type of uncertain disturbance problem, the so-called robust control method can be effectively applied to guarantee the stability of the dynamic systems, while ensuring

Fig. 1. The structure of the metro line.

a smaller prescribed disturbance attenuation level with respect to the uncertain disturbances [3,25]. This is suited for the state-space train traffic model of the metro line systems.

In this research, we will focus on the robust train regulation of metro lines with a stochastic passenger arrival flow within the framework of robust control theory. The contributions of this paper are as follows.

1. By assuming that the stochastic passenger arrival flow is dependent on a discrete Markovian process, a constrained discrete state-space model for the train traffic of metro line systems is developed. The stochastic passenger flow will lead to the instability of the metro line. Based on stochastic stability theory, we present the stochastic stability condition for the train traffic subject to control constraints.
2. For the uncertain disturbances leading to the delays of train traffic, we define an H_∞ disturbance attenuation level γ as the robustness of the deviations of the actual timetable from the nominal schedule. The existence condition for the robust train regulation strategy is given in terms of linear matrix inequalities theoretically, and can be easily solved by Matlab LMI Tools.
3. By regarding the H_∞ disturbance attenuation level as the optimization performance, a nonlinear optimization problem is formulated to determine the optimal robust train regulation strategy that guarantees the minimized disturbance attenuation level to reduce the total train delays. To solve this optimization problem, we design an effective iterative algorithm to generate the robust train regulation strategy with a smaller H_∞ disturbance attenuation.

The rest of this paper is organized as follows. In Section 2, the train traffic model with stochastically dynamic passenger arrival flow and uncertain disturbances is presented. In Section 3, the stochastic stability of the metro line is analysed, and the robust train regulation is designed. In Section 4, numerical examples are provided to demonstrate the effectiveness of the proposed methods. We conclude this paper in Section 5.

2. Problem description

Consider a metro line with $N+1$ stations and N sections, and an ordered set of trains are running on the sections and stop at the stations to allow passengers to get on and off. In this paper, we restrict our studies to a metro line with a sequential line structure, in which trains start from the first station and leave the line after station N. The considered structure of the metro line is shown as Fig. 1.

Throughout this paper, the following assumptions are made in order to formulate the problem.

(A1) The running time of a train between two successive stations does not depend on the number of passengers on the train;
(A2) The dwell time of a train at a station is affected by the number of passengers getting on the train;
(A3) The average passenger arrival rate at each station of the metro line is dependent on a discrete Markovian process in a special period (such as peak hours).

Assumption A1 is rational because the running time of a train is mainly affected by the traction force of the train, rather than the number of passengers on the train. Assumption A2 is reasonable because, in practice, the dwell time of the train at the station increases proportionally to the number of passengers getting on the train. In addition, Assumptions A1 and A2 can be replaced by more sophisticated assumptions, for instance, by considering the effect of the number of passengers getting off the train on the dwell time of the train. However, these more sophisticated assumptions are not significantly different as far as train regulation analysis is concerned. For Assumption A3, we choose a Markovian process to describe the dynamic changing and random characteristic of the passenger arrival flow. To prove the rationality of this assumption, we collected practical data on the passenger arrival flow of one station on the Yizhuang metro line for the morning peak hours and proved the rationality of this assumption via the method of χ^2 hypothetical testing for the discrete-time Markovian process in the following numerical examples. In addition, a Markovian model for describing the dynamic changing of the passenger arrival flow will reduce the conservativeness of the stability condition of the metro line system. If a non-Markovian model, such as an arbitrary switching model, is used for the passenger average arrival rate, a more conservative condition will be obtained for the stability of the metro line system.

For the operation of train traffic in metro line, the symbols and parameters are listed below.

$i = 1, 2 \ldots, M$: indices of trains on the line;

$k = 1, 2, \dots, N$: indices of stations on the line;
t_k^i: the departure time of the i-th train from the k-th station;
r_k^i: the running time of the i-th train from the k-th station to $k+1$-th station;
s_k^i: the dwell time of the i-th train at the k-th station;
R_k^i: the nominal running time of the i-th train from the k-th station to the $k+1$-th station;
D_k: the minimal dwell time at a station when no passenger gets on the train;
T_b: the buffer time of the train at each station.

2.1. The train traffic model of metro lines

To study the train regulation problem, this paper first considers a given nominal timetable. Let a nominal timetable T_k^i be the departure time of each train i at each station k. Then, we have the following transfer equation.

$$T_{k+1}^i = T_k^i + R_k^i + a_{k+1}\lambda_{k+1}(T_{k+1}^i - T_{k+1}^{i-1}) + D_{k+1} + T_b, \tag{1}$$

where a_k is the average boarding time of per passenger at the station, R_k^i is the nominal running time of the i-th train from the k-th station to the $k+1$-th station, λ_k is the passengers average arrival rate of the time interval, and the unit is the number of passengers per second, D_{k+1} is minimal dwell time of the train at station $k+1$ and T_b is the buffer time of the train at station $k+1$. The nominal timetable is characterized by a constant time interval H between two successive trains, i.e., $H = T_{k+1}^i - T_{k+1}^{i-1}$. The transfer Eq. (1) can be further rewritten as

$$(1 - a_{k+1}\lambda_{k+1})T_{k+1}^i = T_k^i + R_k^i - a_{k+1}\lambda_{k+1}T_{k+1}^{i-1} + D_{k+1} + T_b. \tag{2}$$

Moreover the practical departure time of the i-th train from the $k+1$-th station is given as

$$t_{k+1}^i = t_k^i + r_k^i + s_{k+1}^i, \tag{3}$$

which shows that the departure time of the i-th train from the $k+1$-th station is determined by the running time r_k^i of the i-th train from k-th to $k+1$-th station and the dwell time s_{k+1}^i of the i-th train at station $k+1$.

By Assumption A1, the running time of the i-th train from k-th to $k+1$-th station is formulated as

$$r_k^i = R_k^i + {u_1}_k^i + {w_1}_k^i, \tag{4}$$

where ${u_1}_k^i$ is the control strategy used to magnify the running time of the i-th train between the k-th and $k+1$-th stations, which is used to increase the running time when ${u_1}_k^i > 0$ and decrease the running time when ${u_1}_k^i < 0$. ${u_1}_k^i$ is subject to speed limit and safety requirement constraints. ${w_1}_k^i$ is the uncertain disturbance term in the running time, which is supposed to be of finite energy, i.e., $\sum_{k=1}^{\infty} {w_1}_k^i {w_1}_k^i < \infty$.

According to Assumptions A2–A3, the dwell time increases proportionally to the number of passengers getting on the train. Thus, the dwell time depends on the passenger arrival rate and the time interval between the departure of the preceding train and the arrival of the current train, which is modelled as

$$s_k^i = a_k\lambda_k(t_k^i - t_k^{i-1}) + D_k + {u_2}_k^i + {w_2}_k^i, \tag{5}$$

where $t_k^i - t_k^{i-1}$ is the time interval between the departure of the preceding train and the arrival of the present train, ${u_2}_k^i$ is the dwell time adjustment on train i at station k, which is subject to constraints, and ${w_2}_k^i$ is the uncertain disturbance term to the dwell time, which is also finite energy.

The proposed form of the dwell time (5) is different from the existing results in [22,27], which further considers the dynamically stochastic characteristic of the passengers arrival rates. At different operation stages of the stations in metro lines, the average passenger arrival rate is dynamically changing and dependent on discrete-time Markovian process, which leads to the fact that the dwell time of the train is changing randomly at different operation stages of the stations. In addition, for the passenger arrival rate λ_k, we can guarantee a maximum allowance passenger arrival rate by limiting the maximum value of λ_k to satisfy the limited capacity of the train for carrying passengers.

To investigate the train regulation problem using control theory conveniently, we develop a state-space formulation for the train traffic model. Combining (3)–(5), a state-space representation for the train traffic model is described as

$$(1 - a_{k+1}\lambda_{k+1})t_{k+1}^i = t_k^i - a_{k+1}\lambda_{k+1}t_{k+1}^{i-1} + D_{k+1} + R_k^i + {u_1}_k^i + {u_2}_{k+1}^i + {w_1}_k^i + {w_2}_{k+1}^i. \tag{6}$$

The train traffic model (6) describes the practical local dynamic characteristic of the traffic behaviour related to two successive trains and two successive stations, as shown in Fig. 2. To satisfy the traffic security requirements, overtaking of trains is not allowed. Moreover, the proposed train traffic model (6) involves both the train traffic flow and the passenger flow. It should be noted that the proposed train traffic model (6) involves both the i-th and the $i-1$-th trains, where the departure time of train i at station $k+1$ is affected not only by its departure time at station k, but also by the departure time of its preceding train $i-1$ at station $k+1$. According to the proposed train traffic model, if one train is delayed, the following trains will also be affected, and the train delay will be potentially propagated to all the other following trains. Thus, the train traffic can affect all the other trains, which indicates the interconnection characteristics of the train traffic

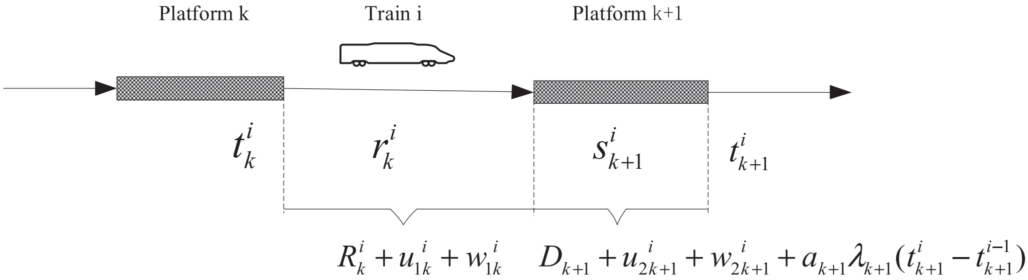

Fig. 2. The illustration of the traffic transition equation (6).

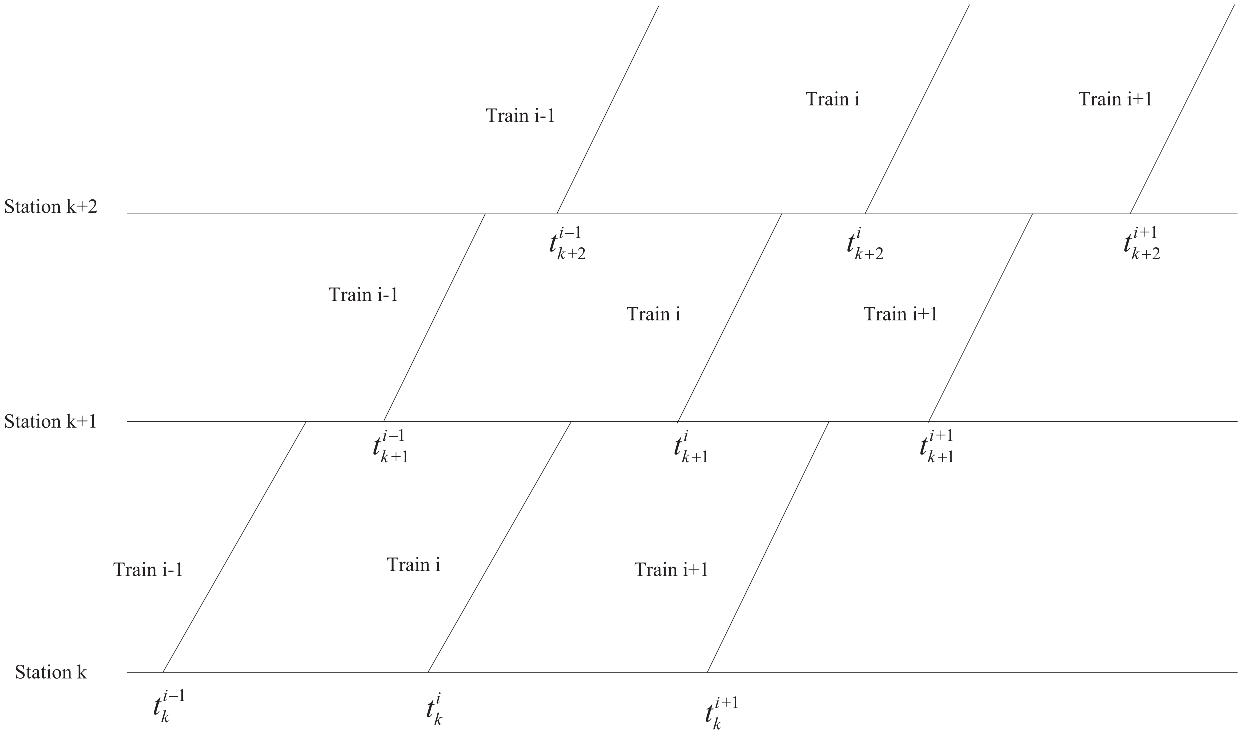

Fig. 3. The illustration of the train traffic flow of the metro line.

flow for metro lines. This situation also coincides with the practical operations of high-frequency metro lines. In addition, to prevent the collision among trains, similar to the assumptions of safety constraints given for the train traffic model by Van Breusegem et al. and Lin and Sheu [22,27], we also consider the assumptions of safety constraints to prevent the train collision for the train traffic model (6). (1) At the initial stage, all the trains keep the safety headway distance. (2) The admission control actions and disturbances are bounded in order to always satisfy the security requirement to prevent collisions among trains.

To demonstrate the evolution of the train traffic flow clearly, the illustration for the train traffic flow is plotted in Fig. 3, which indicates that the departure time of each train for the train traffic flow transfers from one station to the next in metro lines, and any two neighbouring trains ensure a safety distance to satisfy traffic security requirements.

To guarantee that the trains are operating according to the nominal timetable, we introduce x_k^i as the deviation variable of the actual departure time t_k^i from the nominal value T_k^i, i.e., $x_k^i = t_k^i - T_k^i$. Then, by subtracting Eq. (2) from (6), one can obtain the error state-space model for the transfer of the i-th train as follows.

$$(1 - a_{k+1}\lambda_{k+1})x_{k+1}^i = x_k^i - a_{k+1}\lambda_{k+1}x_{k+1}^{i-1} + u_k^i + w_k^i, \tag{7}$$

where $u_k^i = {u_1}_k^i + {u_2}_{k+1}^i - T_b$ and $w_k^i = {w_1}_k^i + {w_2}_{k+1}^i$. From the system-theoretic standpoint for the error state-space model (7), it is convenient to apply the discrete dynamic system theory to study the train regulation problem for metro lines.

2.2. Stochastic passenger arrival flow

Under the hypothesis of the random arrival of passengers, at each stage, the passenger arrival flow is assumed to follow a Poissonian distribution with the average passenger arrival rate, and the average passenger arrival rates at different stages are assumed to satisfy a discrete Markovian process. Let $\theta(t)$ be a Markovian chain taking values in a finite state space $S = 1, 2, \dots, s$ with probability transition matrix $\Pi = (\pi_{mn})_{s\times s}$ given by

$$\Pr\{\theta(t+1) = n|\theta(t) = m\} = \pi_{mn}, \ \forall m, n \in S, \tag{8}$$

where $\pi_{mn} \geq 0$, $\sum_{n=1}^{s} \pi_{mn} = 1$, and t represents the different stages.

Within the framework of the discrete Markovian process, the dwell time s_k^i can be rewritten as

$$s_k^i(\theta(t)) = a_k\lambda(\theta(t))(t_k^i - t_k^{i-1}) + D_k + w2_k^i, \tag{9}$$

where $\theta(t)$ is the stochastic switching mode at stage t, which shows that the dwell time $s_k^i(\theta(t))$ is dynamically and stochastically switching at different stages of the stations with a discrete Markovian jumping parameters. To simplify the analysis, we assume here that the passenger arrival rate at each station is characterized by the same probability transition matrix. Metro lines are well known to be naturally unstable, and the stochastic switching of the dwell time will aggravate the delays of trains with respect to the nominal timetable. To address this problem, the definition of the stochastic stability of the discrete Markovian system is given as follows [11].

Definition 2.1. Consider the following discrete time Markovian system

$$x_{t+1} = H(\theta(t))x_t, \tag{10}$$

where $\theta(t)$ is a finite state time-homogenous or time-inhomogeneous Markovian chain with state space S. Let $(\Omega, \mathcal{F}, \mathcal{P})$ denote the underlying probability space and let Φ be the collection of all probability distribution on S. E$\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. The discrete time Markovian system (10) is said to achieve stochastic stability if for any initial condition $x(0)$ and any initial probability distribution $\theta(0) \in \Phi$,

$$\lim_{t\to+\infty} \mathrm{E}\{\|x_t(x(0), \theta(0)\|^2\} = 0. \tag{11}$$

Concerning the instability of metro lines with stochastic passenger arrival flow, Definition 2.1 provides the stochastic stability condition for addressing the instability problem of the train traffic. The reference by Fang and Loparo [11] introduced several necessary and sufficient conditions for stochastic stability in the form of Lyapunov functions. To conveniently apply the Lyapunov functions of modern control theory, we will further define the matrix form of the train traffic model with stochastic dynamics.

2.3. The matrix form of the train traffic model

There are three main train traffic models for metro lines [27]: the stations sequential model(SSM), the train sequential model (TSM), and the real-time model (RTM). Among these three models, real-time model (RTM) is the only model that provides complete on-line feedback control [27]. Therefore, in this paper, we will adopt real time model (RTM) to describe the train operations. According to (7)–(9), we now propose the formulation for the train traffic model based on information propagations considerations, i.e., t_{k+1}^i is generated by t_k^i and t_{k+1}^{i-1} for all trains and stations. Then, the matrix form of the train traffic model with stochastic dynamics can be expressed as

$$t_{k+1} = A(\theta(k))t_k + B(\theta(k))u_k + B(\theta(k))w_k + R + D, \tag{12}$$

where k indexes the stage of the train traffic model, $\theta(k)$ represents the stochastic switching mode at stage k, $t_k = [t_1^{k-1}, t_2^{k-2}, \dots, t_N^{k-N}]^T$, $u_k = [u_0^k, u_1^{k-1}, \dots, u_{N-1}^{k-N+1}]^T$, $w_k = [w_0^k, w_1^{k-1}, \dots, w_{N-1}^{k-N+1}]^T$, $R = [R_0, R_1, \dots, R_{N-1}]^T$, $D = [D_1, D_2, \dots, D_N]^T$, $T_k = [T_1^{k-1}, T_2^{k-2}, \dots, T_N^{k-N}]^T$,

$$A(\theta(k)) = \begin{bmatrix} -\dfrac{c_1(\theta(k))}{1-c_1(\theta(k))} & 0 & 0 & 0 & \cdots \\ \dfrac{1}{1-c_2(\theta(k))} & -\dfrac{c_2(\theta(k))}{1-c_2(\theta(k)} & 0 & 0 & \cdots \\ & \cdots & \cdots & \cdots & \\ 0 & \cdots & 0 & \dfrac{1}{1-c_N(\theta(k))} & -\dfrac{c_N(\theta(k))}{1-c_N(\theta(k)} \end{bmatrix}_{N\times N},$$

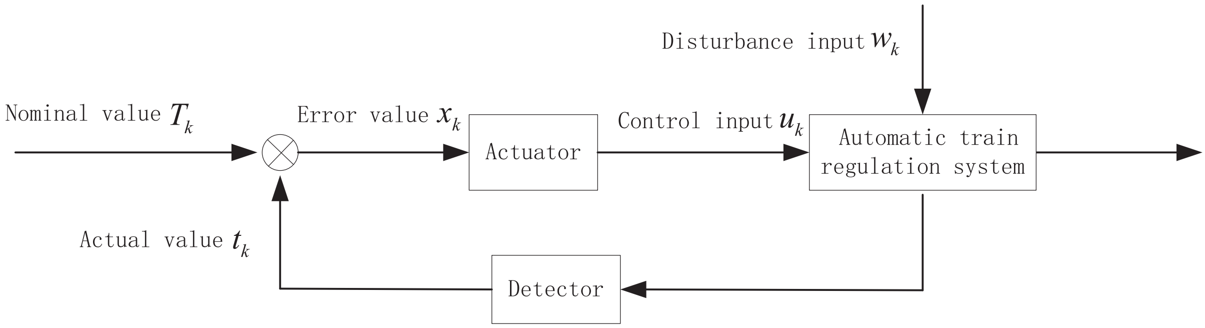

Fig. 4. The diagram of automatic train regulation system.

$$B(\theta(k))=\begin{bmatrix} \frac{1}{1-c_1(\theta(k))} & 0 & 0 & \cdots \\ 0 & \frac{1}{1-c_2(\theta(k))} & 0 & \cdots \\ & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \frac{1}{1-c_N(\theta(k)} \end{bmatrix}_{N\times N}, \quad c_j(\theta(k))=a_j\lambda(\theta(k)),\ \ j=1,2\ldots,N.$$

In addition, it should be noted that according to the original error state-space model (7) for the transfer of each train, we can observe that the coefficients of u_k^i and w_k^i are equal. Thus, by converting the original error state-space model (7) to the matrix form of the train traffic model (12), the system parameter $B(\theta(k))$ for u_k and w_k still take the same form.

According to the matrix form of the train traffic model (12), it is obvious that the system dimension is N, which is equivalent to one less than the number of the stations and is not related to the number of trains. Moreover, the matrix form of the error state-space model for the transfer of the i-th train from the k-th to $k+1$-th station is obtained as

$$x_{k+1}=A(\theta(k))x_k+B(\theta(k))u_k+B(\theta(k))w_k, \tag{13}$$

where $x_k=[x_1^{k-1},x_2^{k-2},\ldots,x_N^{k-N}]^T$, which for notation simplicity is denoted as $x_k\triangleq[x_{k1},x_{k2},\ldots,x_{kN}]^T$.

Under the framework of the state-space formulation, the error departure time x_k is the state variable, the adjustment of the running time and dwell time u_k is the control input variable, and the uncertain event w_k is the disturbance. $A(\theta(k))$ and $B(\theta(k))$ are the system parameters. The automatic train regulation design problem is formulated as a control system design problem. The control diagram of the automatic train regulation system is plotted in Fig. 4. The goal is to design the control input u_k such that the practical train timetable tracks the nominal timetable with respect to the uncertain disturbances to reduce the total train delays.

Remark 2.1. The error state-space model (13) is a stochastic discrete dynamic system, which describes the dynamic evolution of the deviations of the actual timetable from the nominal timetable. The problem of the actual train timetable tracking the nominal timetable is then converted into the stability problem of a stochastic discrete dynamic system. It is convenient to study the train regulation problem using the stability theory of the stochastic discrete dynamic system. Furthermore, for the uncertain disturbances leading to the delays, based on the robust control theory, we will present the robust train regulation problem for metro lines in the next section.

2.4. The robust train regulation problem

In practice, the instability of metro lines will be amplified with time and the operation of other trains will be disturbed, leading to the deviations of the trains from the nominal timetable and reducing the train operation efficiency. To improve the train operation efficiency, we would like the controller to force the practical train timetable to track the nominal timetable by rejecting the effect of the uncertain disturbances to reduce the total train delays. To address this, we choose the robust state feedback control for the train regulation as

$$u_k=K(\theta(k))x_k, \tag{14}$$

where $K(\theta(k))$ is the control parameter to be determined. It shows that the control form (14) is an on-line state feedback control that is implementable: the control to be applied to the i-th train between the k-th and $k+1$-th stations is a linear combination of deviations x_α^β with $\alpha+\beta=i+k$. These deviations are known nearly simultaneously, which enables real-time practical implementation of a state feedback control policy. In practical operations, at each stage k, we can easily obtain the practical departure time t_k of trains using real-time monitoring technology. For the given nominal departure time T_k, we can further calculate the error departure time $x_k = t_k - T_k$, i.e., the delay feedback information of the trains at stage k. Then, based on the train delay feedback information x_k at stage k, we can calculate the value of the controller u_k according to the proposed train regulation algorithm in the next section. The obtained value of controller u_k is then used to regulate the running time and dwell time of trains according to the train model (13) to recover train delays. Because the control form (14) involves all stopping times at all stations, it is a global control. Moreover, it is shown that the proposed regulation strategy for train i at station k, where $i+j=k+1$, is based on all the time deviation information x_k^i relative to index $i+k=j$, which will efficiently provide coordinated control of all the trains running on the metro lines.

Consider that the regular linear quadratic regulator (LQR) formulation cannot be used for a system with uncertain disturbances. To study the problem of the practical timetable tracking the nominal timetable with respect to the uncertain disturbances, we adopt the H_∞ control theory to study the robust train regulation problem. H_∞ control guarantees not only that the system is stable, but also that the system has a smaller prescribed H_∞ disturbance attenuation level [3,20]. According to the H_∞ control theory, the robust train regulation problem in metro lines in this paper can be formulated as the following definition.

Definition 2.2. For the error state-space model (13), given a prescribed H_∞ disturbance attenuation level $\gamma > 0$, obtain the control gain $K(\theta(k))$ such that the error state-space model (13) with $w_k = 0$ is stochastically stable, and the following condition holds

$$\mathrm{E}\left\{\sum_{k=k_0}^{\infty} x_k^T x_k\right\}^{1/2} \leq \gamma \left(\sum_{k=k_0}^{\infty} w_k^T w_k\right)^{1/2} \tag{15}$$

under the zero initial condition for any nonzero w_k with finite energy, where k_0 is the initial stage.

According to the H_∞ theory, the robustness of the deviations of the actual timetable from the nominal timetable is measured by an H_∞ disturbance attenuation level γ. Then, the objective function is a desirable robustness performance in terms of the H_∞ measure, which is used for reducing the total train delays. Moreover, by Definition 2.2, we can also get that

$$\frac{\mathrm{E}\left\{\sum\limits_{k=k_0}^{\infty} x_k^T x_k\right\}^{1/2}}{\left(\sum\limits_{k=k_0}^{\infty} w_k^T w_k\right)^{1/2}} \leq \gamma, \tag{16}$$

which shows that the robustness performance γ is an upper bound of the proportion of the accumulated delays to the accumulated disturbances. Thus, γ can be regarded as a robustness measure of the effect of the uncertain disturbances on train delays. By minimizing the value of γ, one can minimize the accumulated train delays under the maximum allowable uncertain disturbances to improve the robustness of the train regulation strategy. However, the commonly used objective function only includes the accumulated train delays. By comparison, the adopted γ is used to improve the robustness of the train regulation strategy while reducing the accumulated train delays under all allowable disturbances, thereby improving the previous objective function, which is only for reducing the accumulated train delays. Moreover, the multi-step index of the robustness performance is considered, which is different from that in [27] by adopting the simple one-step-ahead performance index.

Additionally, the control input u_k will be under some constraints for the physical limitations of the actuators and the safety constraints. For this purpose, the control input u_k is assumed to satisfy the following constraints

$$-\check{u}_l \leq u_{kl} \leq \hat{u}_l, \tag{17}$$

where u_{kl} is l-th elements of the control input u_k, and $\check{u}_l$ and $\hat{u}_l$ are known positive constants. Considering the fact the train can go much slower, but not much faster, it holds that $\check{u}_l < \hat{u}_l$.

The goal of robust train regulation is to develop the state feedback control that satisfies all the constraints to track the nominal timetable by rejecting the effect of uncertain disturbances. To solve this problem, we formulate it as an H_∞ control problem and synthesize the state feedback control as the robust train regulation strategy that assures the H_∞ tracking performance.

3. Robust train regulation for metro lines

In this section, based on the stochastic stability theory and robust control method, we will study the robust train regulation for metro lines with the stochastic passenger arrival rate and uncertain disturbances.

3.1. Stochastic stability condition

First, we will present the stochastic stability condition for the metro line under the control constraint when the uncertain disturbance $w_k = 0$ as the following proposition.

Proposition 3.1. *Consider the error state-space model (13) for the metro line with $N+1$ stations and $w_k = 0$. Let k_0 be the initial stage of the metro line. If there exist positive definite matrices $X(i)$, $Z(i)$, and any matrices $Y(i)$, $i = 1, 2, \ldots, s$ with appropriate dimensions such that the following linear matrix inequalities (LMIs) hold*

$$\begin{bmatrix} -X(i) & \Omega_1(i) \\ \Omega_1^T(i) & -\Omega_2(i) \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} 1 & x_{k_0}^T \\ x_{k_0} & X(\theta(k_0)) \end{bmatrix} \geq 0, \tag{19}$$

$$\begin{bmatrix} Z(i) & Y(i) \\ Y^T(i) & X(i) \end{bmatrix} \geq 0, \tag{20}$$

$$Z_{ll}(i) \leq \bar{u}_l^2, \tag{21}$$

where

$$\begin{aligned} \Omega_1(i) &= [\sqrt{\pi_{i1}}(X(i)A^T(i) + Y^T(i)B^T(i)), \sqrt{\pi_{i2}}(X(i)A^T(i) + Y^T(i)B^T(i)), \ldots, \\ &\quad \sqrt{\pi_{is}}(X(i)A^T(i) + Y^T(i)B^T(i))], \\ \Omega_2(i) &= \mathrm{diag}\{X(1), X(2), \ldots, X(s)\}, \end{aligned} \tag{22}$$

$\bar{u}_l = \frac{\hat{u}_l + \check{u}_l}{2} - |\frac{\hat{u}_l - \check{u}_l}{2}|$, and $Z_{ll}(i)$ denotes the l-th diagonal element of the matrix $Z(i)$, then the state feedback control $u_k = Y(i)X^{-1}(i)x_k$ is obtained as the train regulation strategy such that the error state-space model (13) is stochastically stable subject to the control constraint (17).

Proof. For the error state-space model (13) with $w_k = 0$ under the initial condition x_{k_0} and the initial state $\theta(k_0)$, construct the following Lyapunov function candidate

$$V(k) = x_k^T P(\theta(k)) x_k, \ \ k \geq k_0, \tag{23}$$

where $P(\theta(k)) > 0$.

Let the stochastic switching mode at stage k be i, that is $\theta(k) = i$. Recall that at the next stage $k+1$, the system may jump to any mode $\theta(k+1) = j$. One can then obtain that

$$\begin{aligned} \Delta V(k) &= \mathrm{E}\{V(x_{k+1}, \theta(k+1))\} - V(x_k, \theta(k)) \\ &= \mathrm{E}\{x_{k+1}^T P(\theta(k+1)) x_{k+1} | \theta(k) = i\} - x_k^T P(i) x_k \\ &= (A(i)x_k + B(i)K(i)x_k)^T \sum_{j=1}^{s} \pi_{ij} P(j)(A(i)x_k + B(i)K(i)x_k) - x_k^T P(i) x_k \\ &= x_k^T [(A(i) + B(i)K(i))^T \sum_{j=1}^{s} \pi_{ij} P(j)(A(i) + B(i)K(i))] x_k - x_k^T P(i) x_k. \end{aligned} \tag{24}$$

Here it should be pointed that the term $V(x_k, \theta(k))$ at the current stage k is a determinate value and there is no expectation E for the term $V(x_k, \theta(k))$, while at the next stage $k+1$, the system mode may randomly jump to any mode $\theta(k+1) = j (j = 1, 2, \ldots, s)$, so the term $V(x_{k+1}, \theta(k+1))$ at the next stage $k+1$ is a random variable, and the expectation E has been used for the term $V(x_{k+1}, \theta(k+1))$.

In addition, by variable substitution, let $X(i) = \alpha P^{-1}(i)$, $Y(i) = K(i)X(i)$, where $\alpha > 0$. According to Schur Complement [4], pre and post-multiplying both sides of (18) by $\mathrm{diag}\{\alpha^{1/2}X^{-1}(i), \alpha^{-1/2}I, \alpha^{-1/2}I, \ldots, \alpha^{-1/2}I\}$, one can obtain that the inequality (18) is equivalent to the following inequality

$$(A(i) + B(i)K(i))^T \sum_{j=1}^{s} \pi_{ij} P(j)(A(i) + B(i)K(i)) - P(i) < 0.$$

Thus, by (24), we have $\Delta V(k) < 0$ for all stages $k (k \geq k_0)$. According to Definition 2.1 and Lyapunov stability theory, the error state-space model (13) is stochastically stable.

Moreover, by Schur Complement [4], inequality (19) is equivalent to

$$x_{k_0}^T P(\theta(k_0)) x_{k_0} \leq \alpha. \tag{25}$$

Pre and post-multiplying both sides of (20) by diag$\{I, X^{-1}(i)\}$, one can get that (20) is equivalent to

$$\begin{bmatrix} Z(i) & K(i) \\ K^T(i) & \alpha^{-1}P(i) \end{bmatrix} \geq 0. \tag{26}$$

According to $\Delta V(k) < 0$ for all stages $k(k \geq k_0)$, it follows from (24) and (25) that, for all stage $k > k_0$, one has

$$\begin{aligned} V(k) &< V(k_0) \\ &= x_{k_0}^T P(\theta(k_0))x_{k_0} \\ &\leq \alpha. \end{aligned} \tag{27}$$

In addition, the control constraint (17) can be rewritten as $-\frac{\hat{u}_l+\check{u}_l}{2} \leq u_{kl}(i) - \frac{\hat{u}_l-\check{u}_l}{2} \leq \frac{\hat{u}_l+\check{u}_l}{2}$, i.e., $|u_{kl}(i) - \frac{\hat{u}_l-\check{u}_l}{2}| \leq \frac{\hat{u}_l+\check{u}_l}{2}$. Since $|u_{kl}(i) - \frac{\hat{u}_l-\check{u}_l}{2}| \leq |u_{kl}(i)| + |\frac{\hat{u}_l-\check{u}_l}{2}|$, the control constraint (17) can be guaranteed by the inequality $|u_{kl}(i)| \leq \frac{\hat{u}_l+\check{u}_l}{2} - |\frac{\hat{u}_l-\check{u}_l}{2}|$.

Note that

$$\begin{aligned} \| u_{kl}(i) \|^2 &= \| K_l(i)x_k \|^2 = \| K_l(i)P^{-\frac{1}{2}}(i)P^{\frac{1}{2}}(i)x_k \|^2 \\ &\leq K_l(i)P^{-1}(i)K_l^T(i)x_k^T P(i)x_k \\ &\leq K_l(i)P^{-1}(i)K_l^T(i)V(k_0) \\ &\leq K_l(i)P^{-1}(i)K_l^T(i)\alpha, \end{aligned} \tag{28}$$

where $K_l(i)$ represents the l-th row of the matrix $K(i)$.

It is shown from (21) and (26) that

$$\alpha K(i)P^{-1}(i)K^T(i) \leq Z(i), Z_{ll}(i) \leq \bar{u}_l^2, \tag{29}$$

where $\bar{u}_l = \frac{\hat{u}_l+\check{u}_l}{2} - |\frac{\hat{u}_l-\check{u}_l}{2}|$, which implies from (28) that $u_{kl}(i) \leq \frac{\hat{u}_l+\check{u}_l}{2} - |\frac{\hat{u}_l-\check{u}_l}{2}|$, i.e., the control constraint (17) is satisfied.

Therefore, the error state-space model (13) for the metro line with constraint (17) and $w_k = 0$ is stochastically stable, and the state feedback control $u_k = K(i)x_k,\ k \geq k_0$ is obtained as the train regulation strategy. □

Remark 3.1. Proposition 3.1 provides a sufficient condition to choose the proper state feedback control $u_k = K(i)x_k,\ k \geq k_0$ as the train regulation such that the metro line system is stochastically stable under $w_k = 0$. Note that the sufficient conditions (18)–(21) in Proposition 3.1 take the form of linear matrix inequalities. It can be easily solved using an interior-point algorithm. The control to be applied to train i at station k, where $(i + k = j + 1)$, involves all the time deviations x_k^i relative to index $i + k = j$, which will efficiently achieve the coordinated control of all the trains running on the metro line. Additionally, it should be noted that Proposition 3.1 presents the stochastic stability condition for the error state-space model (13), where the number of stations is related to the dimension of this stochastic discrete system, and the departure frequency of the trains on the metro line is related to the time horizon. Consider the fact that the stochastic stability analysis for the stochastic discrete system is based on the long-time horizon. The high frequency of metro lines can ensure a sufficient time horizon for the stochastic stability analysis of this stochastic discrete system. Thus, it is rational for the proposed stochastic stability analysis result of a metro line with high frequency.

3.2. Robust train regulation results

Based on Definition 2.2 and Proposition 3.1, we will design the robust train regulation for metro lines with the stochastic passenger arrival rate and uncertain disturbances. The following theorem will give a sufficient condition for the existence of the robust train regulation strategy.

Theorem 3.1. *Let $\gamma > 0$ be a given constant. For the error state-space model (13) of the metro line with $N + 1$ stations, if there exist a positive scalar α, positive definite matrixes $X(i)$, $Z(i)$, and any matrices $Y(i),\ i = 1, 2, \ldots, s$ with appropriate dimensions such that the following linear matrix inequalities hold*

$$\begin{bmatrix} -X(i) & 0 & \Omega_1(i) & X(i) \\ 0 & -\alpha\gamma^2 I & \alpha\Omega_3(i) & 0 \\ \Omega_1^T(i) & \alpha\Omega_3^T(i) & -\Omega_2(i) & 0 \\ X(i) & 0 & 0 & -\alpha I \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} 1 & x_{k_0}^T \\ x_{k_0} & X(\theta(k_0)) \end{bmatrix} \geq 0, \tag{31}$$

$$\begin{bmatrix} Z(i) & Y(i) \\ Y^T(i) & X(i) \end{bmatrix} \geq 0, \tag{32}$$

$$Z_{ll}(i) \leq \bar{u}_l^2, \tag{33}$$

where $\Omega_1(i)$, $\Omega_2(i)$ take the same forms of (21), $\Omega_3(i) = [B^T(i), B^T(i), \ldots, B^T(i)]$, and $\bar{u}_l = \frac{\hat{u}_l + \check{u}_l}{2} - |\frac{\hat{u}_l - \check{u}_l}{2}|$, then the robust state feedback control $u_k = Y(i)X^{-1}(i)x_k$, $k \geq k_0$ is obtained as the robust train regulation to guarantee that the practical train timetable tracks the nominal timetable with a disturbance attenuation level γ so as to reduce the total train delays.

Proof. First, according to Schur complement [4], it can be derived that the inequality (30) implies that the condition (18) holds. So according to Proposition 3.1, under the conditions (30)–(33), the error state-space model (13) with $w_k = 0$ and control constraints (17) is stochastically stable.

Next for any nonzero disturbance w_k with finite energy, calculating $\Delta V(k)$ yields that

$$\begin{aligned} \Delta V(k) &= \mathrm{E}\{V(x_{k+1}, \theta(k+1))\} - V(x_k, \theta(k)) \\ &= (A(i)x_k + B(i)K(i)x_k + B(i)w_k)^T \sum_{j=1}^{s} \pi_{ij} P(j)(A(i)x_k + B(i)K(i)x_k + B(i)w_k) - x_k^T P(i) x_k \\ &= \begin{bmatrix} x_k \\ w_k \end{bmatrix}^T \begin{bmatrix} -P(i) + \Theta_1(i) & \Theta_2(i) \\ \Theta_2^T(i) & B^T(i) \sum_{j=1}^{s} \pi_{ij} P(j) B(i) \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix}, \end{aligned} \tag{34}$$

where

$$\Theta_1(i) = (A(i) + B(i)K(i))^T \sum_{j=1}^{s} \pi_{ij} P(j)(A(i) + B(i)K(i)), \tag{35}$$

$$\Theta_2(i) = (A(i) + B(i)K(i))^T \sum_{j=1}^{s} \pi_{ij} P(j) B(i). \tag{36}$$

Additionally, let $X(i) = \alpha P^{-1}(i)$, $Y(i) = K(i)X(i)$. Pre and post-multiplying both sides of (30) by $\mathrm{diag}\{\alpha^{1/2} X^{-1}(i), \alpha^{-1/2} I, \alpha^{-1/2} I, \ldots, \alpha^{-1/2} I\}$, we can get condition (30) is equivalent to

$$\begin{bmatrix} -P(i) + \Theta_1(i) + I & \Theta_2(i) \\ \Theta_2^T(i) & -\gamma^2 I + B^T(i) \sum_{j=1}^{s} \pi_{ij} P(j) B(i) \end{bmatrix} < 0. \tag{37}$$

It obviously follows from (34) and (37) that

$$\Delta V(k) + \mathrm{E}\{x_k^T x_k\} - \gamma^2 w_k^T w_k < 0, \tag{38}$$

which implies that

$$\sum_{k=k_0}^{\infty} (\Delta V(k) + \mathrm{E}\{x_k^T x_k\} - \gamma^2 w_k^T w_k) < 0, \tag{39}$$

Thus, noting that $\lim_{k \to +\infty} \mathrm{E}\{x_k\} = 0$, under the zero initial condition $(x_{k_0} = 0)$, we have

$$\mathrm{E}\left\{ \sum_{k=k_0}^{\infty} x_k^T x_k \right\}^{1/2} \leq \gamma \left(\sum_{k=k_0}^{\infty} w_k^T w_k \right)^{1/2}. \tag{40}$$

Therefore, according to Definition 2.2, the robust state feedback control $u_k = Y(i)X^{-1}(i)x_k$, $k \geq k_0$ is obtained as the train regulation to guarantee that the practical train timetable tracks the nominal timetable with a disturbance attenuation level γ to reduce the total train delays. □

Remark 3.2. According to conditions (30)–(33) of Theorem 3.1, the robust state feedback control for the train regulation for metro lines can be obtained such that the practical train timetable is robust to uncertain disturbances with a disturbance attenuation level γ. The proposed method in Theorem 3.2 provides a closed-loop decision-making approach for the train regulation that is easily implementable to improve the robustness of the train regulation with respect to uncertain disturbance with finite energy. Additionally, as an extension, if the disturbance is considered as a random variable satisfying a certain probabilistic distribution, the proposed discrete stochastic system model and robust control method by [17,18] can be well extended to address the train regulation problem for metro lines with stochastic disturbances, which will be a future topic.

Furthermore, from a practical point of view, we often hope that the system has a smaller H_∞ disturbance attenuation γ to effectively reduce the total train delays. Then, under the control constraint (17), the optimal H_∞ control of the robust train regulation is introduced as follows.

$$\min_{(\alpha, X(i), Y(i), Z(i))} \quad \gamma^2 \tag{41}$$

$$\begin{aligned} \text{subject to } & \text{(i) } (30) - (33), \\ & \text{(ii) } \alpha > 0,\ X(i) > 0, Z(i) > 0,\ i = 1, 2 \ldots, s. \end{aligned} \tag{42}$$

Note that, if γ is regarded as a variable, the term $\alpha\gamma^2$ in condition (30) is nonlinear, i.e., the obtained sufficient condition for the existence of an optimal robust train regulation strategy is nonlinear and not the form of a linear matrix inequality. Then, compared to common robust control problems, the difficulty we face is that the obtained sufficient condition cannot be directly solved using the interior-point algorithm. To solve this nonlinear optimization problem, we design an effective iteration algorithm along with the interior-point algorithm to generate the state-feedback control for the train regulation strategy with a less conservative H_∞ disturbance attenuation, which further enriches the robust control theory and is presented as follows.

Algorithm 3.1.

- Step 1. Choose a sufficiently large initial $\gamma > 0$ such that there exists a feasible solution to linear matrix inequalities (LMIs) (30)–(33) and (ii) in (42).
- Step 2. For a given $\gamma > 0$, solve the LMIs (30)–(33) and (ii) in (42) by an interior-point algorithm, which can be implemented with the Matlab LMI Toolbox.
- Step 3. If the LMIs (30)–(33) and (ii) in (42) are infeasible, then exit and output $\gamma = \gamma + \Delta\gamma$ and the corresponding solutions. Otherwise, decrease the positive scalar γ with the step size $\Delta\gamma$, i.e., $\gamma = \gamma - \Delta\gamma$ and go back to Step 2.

Remark 3.3. In the proposed algorithm 3.1, if the positive variable γ is given, then all the constraint conditions in (30)–(33) take the form of linear matrix inequalities, which can be directly solved by the Matlab LMI Toolbox, which implements an interior-point algorithm. For the finite iteration times of the variable γ, the time complexity of algorithm 3.1 is mainly dependent on the scale of the LMIs (30)–(33). Noting that the scale of the LMIs (30)–(33) is related to the number of stations, the time complexity of the proposed algorithm is dependent on the number of stations but not on the number of trains. In particular, for a practical metro line with a dozen stations, the proposed method has a low computational complexity because the LMI-based optimization problems can be solved in polynomial time. Therefore, the proposed algorithm can be effectively and quickly implemented for a practical metro line in real time. Additionally, the proposed method can be extended to address both the case in which the time intervals between successive trains for the stages are different and the case in which the passenger arrival rate at each station takes on different probability transition matrixes.

4. Numerical examples

In this section, to illustrate the validity of the theoretical results proposed in this paper, we will apply our proposed methods to the actual Beijing Yizhuang metro line, which consists of 14 stations (i.e., $N = 13$). The geometric layout of the actual Beijing Yizhuang metro line is plotted in Fig. 5. For the accumulated passenger arrival flow to the train delays, it is necessary to study the robust train regulation during peak hours. Based on this, we perform our method for the metro line during the morning peak hours (from 7:00 a.m. to 8:30 a.m.). The considered nominal running time is between 60 s and 100 s, and the buffer time t_b is set as 15s for each station. In addition, without loss of generality, we consider the case in which the passenger arrival flow and Markovian switching rates for each station are the same, and the average boarding time per passenger for each station is given as $a_k = 0.05$ s.

First, we collect the practical data of the passenger arrival flow of Xiaohongmen station on the Yizhuang metro line during the morning peak hours from 7:30 a.m. to 8:30 a.m. for five working days, as presented in Table 1. From Table 1, we find that there are three approximate switching modes for the average passenger arrival rates during the morning peak hours: 0.3, 0.4, and 0.5. Under the assumption of a Markovian process, we use mode 1, 2 and 3 to denote the three switching values 0.3, 0.4, and 0.5, respectively. Moreover, according the practical data in Table 1, the probability transition matrix for the three switching modes is calculated as follows.

$$\Pi = \begin{bmatrix} 0.60 & 0.25 & 0.15 \\ 0.47 & 0.33 & 0.20 \\ 0.40 & 0.40 & 0.20 \end{bmatrix}. \tag{43}$$

To verify the rationality of this assumption, we adopt the χ^2 hypothetical test method for the discrete time Markovian process. First, the χ^2 statistic is chosen as $\chi^2 = 2\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}\ln\frac{p_{ij}}{p_j}$ [19], where m represents the total number of switching

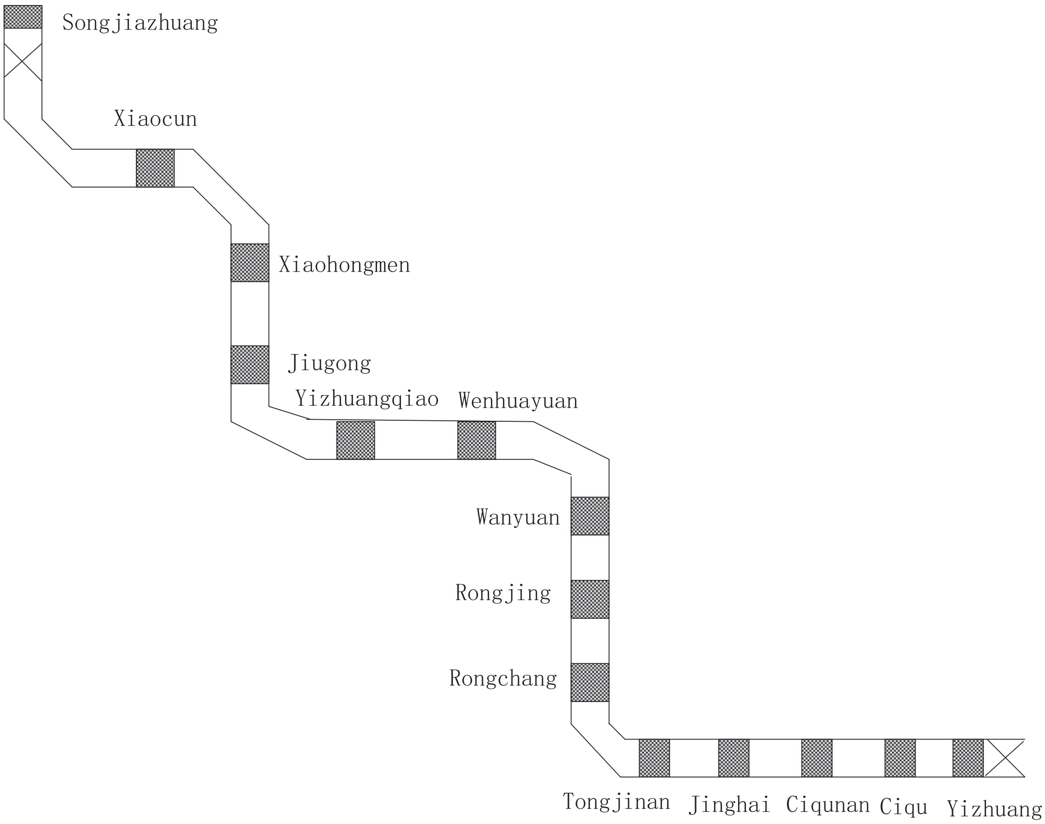

Fig. 5. Beijing Yizhuang metro line map.

modes, f_{ij} is the total number of transitions from state i to state j, p_{ij} is the transition probability from state i to state j, and $p_j = \frac{\sum_{i=1}^{m} f_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}}$ is the marginal probability. Then, according to the practical data in Table 1, we calculate that $\chi^2 = 13.831$. For the given significant level $\alpha = 0.05$, $\chi^2_\alpha(4) = 9.488$. Because $\chi^2 > \chi^2_\alpha(4)$, we can find that the assumption that the passenger arrival rate is dependent on a discrete time Markovian process is rational. Under the probability transition matrix (43), we can design the robust train regulation to suppress the effect of the stochastic passenger arrival flow and the uncertain disturbances to the nominal timetable of the metro line. To show the robustness and the stochastic stability of the proposed robust train regulation strategy, we will consider three scenario studies with larger train delays: Scenario 1 is used to verify the validity of the proposed train regulation model while demonstrating the merit of the proposed method in comparison to the common method using the buffer time, Scenario 2 is used to illustrate the robustness of the proposed method for a metro line with frequent disturbances, and Scenario 3 is used to verify the stochastic stability of the metro line under the proposed method with different stochastic switching modes.

4.1. Scenario 1: one train is affected along all stations

In this scenario, suppose that at time $7:00$ a.m., train 10 is departing from the first station. Because of the effect of uncertain disturbances, train 10 is assumed to be delayed by $[70\ \text{s}, 16\ \text{s}, 18\ \text{s}, 15\ \text{s}, 15\ \text{s}, 16\ \text{s}, 17\ \text{s}, 15\ \text{s}, 15\ \text{s}, 15\ \text{s}, 16\ \text{s}, 17\ \text{s}, 15\ \text{s}]$ from station 1 to station 13. Clearly, train 10 is affected by a relatively larger delay in the first station. For the larger delay 70 s at the first station, the affected train 10 will need several stations to compensate for the delays. Then, we will apply the proposed robust train regulation method to recover the delays of the affected train 10.

According to the probability transition matrix (43), using Monte Carlo simulations, one of the realizations of discrete time Markovian jumping modes for the passenger arrival rates from $7:00$ a.m. to $8:30$ a.m. is shown as Fig. 6. First, under the larger delay, for the error state-space model of train traffic without control, i.e., $u_k = 0$, the error state evolution trend for the affected train 10 is plotted as the square line in Fig. 7, which shows that the stochastic passenger arrival flow and the uncertain disturbances lead to the delays of the departure time of train 10 at all stations. The delays of the affected train 10 follow an increasing trend along the stations Moreover, let t_i^{10} be the departure time of train 10 at station i. The corresponding values of the delays along all the stations are calculated in the first line of Table 2, which shows that within the framework of the common train regulation model using buffer time, the delays of the affected train 10 increase from

Table 1
The passenger arrival flow of Xiaohongmen station in Yizhuang metro line.

	Arrival time	Departure time	Passenger number	Arrival rate	Mode
Day 1	7:33:35	7:34:05	155	0.3	1
	7:40:00	7:40:20	178	0.5	3
	7:46:45	7:47:15	142	0.3	1
	7:52:50	7:53:40	141	0.4	2
	7:59:50	8:00:30	178	0.4	2
	8:06:00	8:06:15	121	0.3	1
	8:12:50	8:13:20	120	0.3	1
	8:19:00	8:19:20	134	0.4	2
	8:25:30	8:26:00	114	0.3	1
	8:32:05	8:32:30	109	0.3	1
Day 2	7:33:55	7:34:10	108	0.3	1
	7:39:55	7:40:20	124	0.3	1
	7:46:45	7:47:35	165	0.4	2
	7:52:45	7:53:40	179	0.5	3
	7:59:35	8:00:20	180	0.5	3
	8:06:15	8:06:55	153	0.4	2
	8:12:45	8:13:10	136	0.4	2
	8:19:00	8:19:35	141	0.4	2
	8:26:10	8:26:40	117	0.3	1
	8:32:15	8:32:35	109	0.3	1
Day 3	7:33:40	7:34:40	154	0.5	3
	7:40:10	7:40:30	140	0.4	2
	7:46:40	7:47:05	146	0.4	2
	7:53:10	7:53:30	176	0.5	3
	7:59:10	7:59:30	162	0.5	3
	8:05:55	8:06:25	138	0.3	1
	8:12:35	8:13:05	183	0.5	3
	8:19:00	8:19:20	120	0.3	1
	8:25:20	8:25:40	109	0.3	1
	8:32:00	8:32:20	104	0.3	1
Day 4	7:33:30	7:33:50	155	0.5	3
	7:40:00	7:40:40	134	0.3	1
	7:46:35	7:46:50	161	0.4	2
	7:53:20	7:53:45	169	0.4	2
	7:59:20	7:59:45	165	0.5	3
	8:06:05	8:06:25	175	0.4	2
	8:12:35	8:13:00	129	0.3	1
	8:18:45	8:19:10	118	0.3	1
	8:25:40	8:26:00	109	0.3	1
	8:31:40	8:32:05	96	0.3	1
Day 5	7:33:35	7:33:55	132	0.4	2
	7:40:25	7:40:45	127	0.3	1
	7:46:40	7:47:05	136	0.4	2
	7:53:00	7:53:15	129	0.3	1
	8:00:10	8:00:25	210	0.5	3
	8:06:00	8:06:25	129	0.4	2
	8:12:50	8:13:10	138	0.3	1
	8:19:00	8:19:20	102	0.3	1
	8:25:45	8:26:05	114	0.3	1
	8:32:25	8:32:45	112	0.3	1

70 s at the first station to 87 s at the last station. The delays of the affected train 10 are propagated from one station to the next station, i.e., the deviation with respect to the nominal timetable is amplified over time. This unstable behaviour is quite uncomfortable for passengers.

Next, according to the proposed robust train regulation model and method in this paper, we design the robust train regulation to suppress the effect of the stochastic passenger arrival flow and the uncertain disturbances to the nominal timetable of the metro line. Suppose that the state feedback control u_k is subject to the constraint $-30\text{s} \le u_k \le 35\text{s}$, i.e., the maximum increase in the running time and dwell time for each train is not allowed to exceed 35 s, and the maximum decrease in the running time and dwell time is not allowed to exceed 30 s. Then, performing Algorithm 3.1 with Matlab LMIs Toolbox, the positive scalar γ decreases with the step size $\triangle\gamma = 0.1$ at each time, the final smaller H_∞ disturbance attenuation is obtained as $\gamma = 16.4$, and the corresponding controller gain can be obtained. To keep the paper concise, the controller gain $K \in R^{13 \times 13}$ is not presented here because the dimension is very high. In particular, the robust controller u_i^{10} is calculated in the last line of Table 2, which shows that the adjustment satisfies the control constraints. In addition, because of the larger delay at the first station, the adjustment is larger at the beginning of the stations and then deceases in

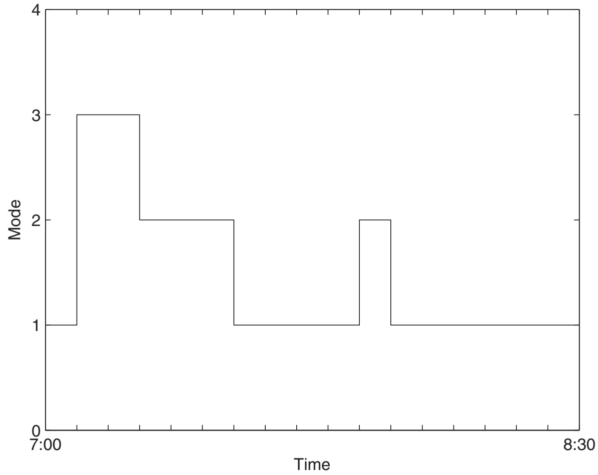

Fig. 6. The realization of discrete time Markovian jumping mode.

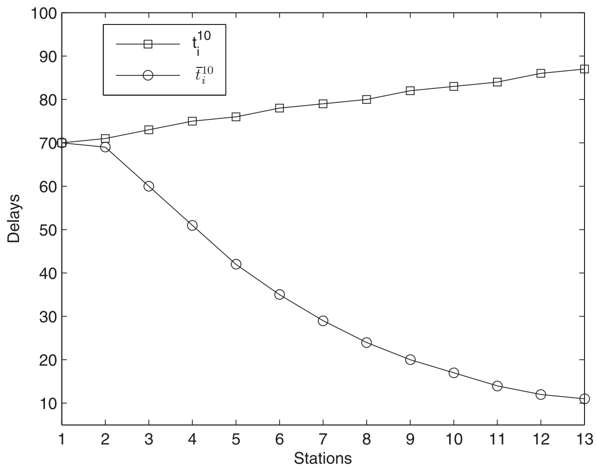

Fig. 7. The delays of the affected train 10.

Table 2
The delays and controller of the affected train 10 at all stations.

Station i	1	2	3	4	5	6	7	8	9	10	11	12	13
t_i^{10}	70 s	71 s	73 s	75 s	76 s	78 s	79 s	80 s	82 s	83 s	84 s	86 s	87 s
$\bar{t}_i^{10}$	70 s	69 s	60 s	51 s	42 s	35 s	29 s	24 s	20 s	17 s	14 s	12 s	11 s
u_i^{10}	−17 s	−26 s	−26 s	−25 s	−22 s	−22 s	−21 s	−20 s	−19 s	−18 s	−17 s	−16 s	null

the following stations. Under the robust train regulation, the error state evolution for the affected train 10 is plotted as the round line in Fig. 7. In contrast to the common train regulation model using buffer time, for which the buffer time allocation is static, the proposed train regulation model dynamically adjusts both the running time and the dwell time of each train in real time using the state-feedback information, thereby reducing the potential redundant buffer time and improving the system capacity utilization. By comparing the square and round lines in Fig. 7, we can observe that under the proposed train regulation model, the delays of train 10 follow a decreasing trend, and the error state of the departure time of train 10 from the nominal timetable is effectively reduced, indicating the validity of the proposed train regulation model. Let $\bar{t}_i^{10}$ be the departure time of train 10 at station i under the robust train regulation. The corresponding values of the delays along all the stations are calculated in the second line of Table 2. From Table 2, we can observe that the delays of train 10 are effectively reduced at all stations and decreased from 70 s at the first station to 11 s at the last station. The proposed robust train regulation method prevents the accumulation of delays for the affected trains along the stations.

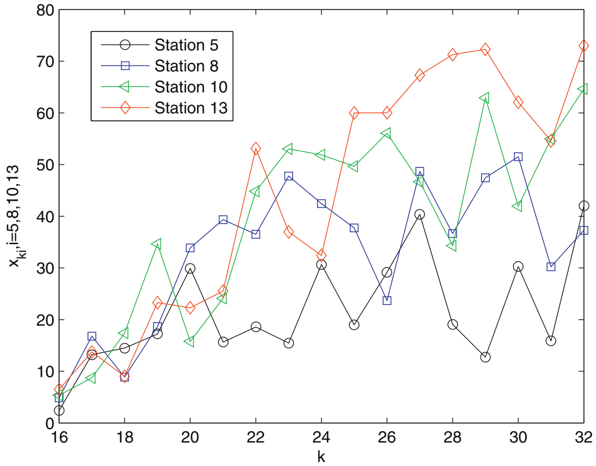

Fig. 8. The error state evolution for the metro system without robust train regulation.

4.2. Scenario 2: multiple trains are affected along all stations

In this case, we consider that there are multiple trains affected by the uncertain disturbances along all stations to demonstrate the robustness of the proposed method for metro lines with frequent disturbances. We also take the Beijing Yizhuang metro line as an example. Assume that the stochastic switching of the passenger arriving rates is also the realization of the discrete-time Markovian jumping mode in Fig. 6. In addition, we choose the uncertain disturbances to the running time and dwell time of trains as a Gaussian white noise with a mean of 20s and standard of 10s, which are larger than the white noise adopted by Van Breusegem et al. [27].

The time horizon is also considered from $7:00$ a.m. to $8:30$ a.m. during the morning peak hours. Suppose that the initial stage for the error state-space model at $7:00$ a.m. is $k_0 = 16$ and that the initial conditions of the error state of the metro line are chosen randomly from the interval [1,10]. Under the stochastic passenger arrival flow and the uncertain disturbances, the error state evolution of the departure time of trains in the metro line at stations 5, 8, 10 and 13 without robust train regulation is plotted in Fig. 8, which shows that the delays of each train are amplified with increasing number of stations because of the accumulated effect of the stochastic passenger arrival flow and uncertain disturbances. The delays are continuously increasing progressively along the stations, which reduces the efficiency of the metro line operation. In particular, for the last station, the maximum delay is up to 72 s. Thus, for larger scale metro lines, the delays of each train will be amplified more seriously along the stations. Therefore, it is necessary to apply the robust train regulation to suppress the delay propagation along large-scale metro lines.

In practice, suppose that the state feedback control u_k is subject to the constraint $-30\ \text{s} \le u_k \le 35\ \text{s}$. By applying the robust train regulation to each train and solving the optimization problem (41), a smaller H_∞ disturbance attenuation γ is also calculated as $\gamma = 16.4$, and the corresponding controller gain can be obtained. Under the robust train regulation, the simulation results of the error state evolution for stations 5, 8, 10 and 13 are plotted in Fig. 9, which shows that the deviations from the nominal timetable for each train are effectively controlled in the range of 22 s. By comparingFig. 8 and Fig. 9, it is obvious that the proposed robust train regulation significantly reduces the delays of all the trains at each station. For the train operation at the last station, when the train operation is without train regulation, the train delay increases from 5 s to 72 s. This delay negatively influences the passengers traveling and reduces the train operation efficiency. By comparison, under the robust train regulation strategy, the delays of the trains at the last station are controlled in a reasonable range of 22 s, which is a accepted level.

Moreover, let w, x, and y represent the overall disturbances, the overall delays without robust train regulation, and the overall delays with robust train regulation of all the trains at each station, respectively. Then the corresponding overall disturbances and delays for all the trains at the 13 stations are calculated as summarized in Table 3. From the third column in Table 3, we can observe that the delays of all the trains increase from one station to the next because of the uncertain disturbances, which shows that the delays are amplified along the stations. Under the robust train regulation, the proportion of the controlled delays to the input disturbances is effectively controlled in the range [0.7, 3.0] (see the fifth column of Table 3), which is substantially less than the H_∞ disturbance attenuation level $\gamma = 16.4$. Therefore, the bullwhip effect of the uncertain disturbances to the train delays is extremely reduced, and the robustness of the train traffic operation is effectively improved. As a result, the overall delays of the trains at each station are reduced to 25%–73% compared to the case without train regulation, which are presented as the last column of Table 3. The evolution trend of the overall delays of the trains along the stations is plotted in Fig. 10, which clearly shows that the robust train regulation strategy significantly

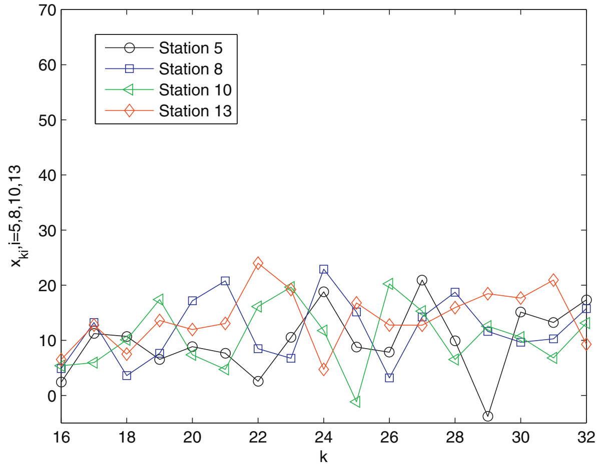

Fig. 9. The error state evolution for the metro system under robust train regulation.

Table 3
The overall disturbances and delays of all the trains at each station.

	w	x	y	y/w	$(x-y)/x$
Station 1	71	72	51	0.71	29.14%
Station 2	100	169	126	1.26	25.37%
Station 3	75	238	142	1.88	40.39%
Station 4	83	306	157	1.90	48.46%
Station 5	95	366	168	1.77	54.00%
Station 6	82	407	159	1.93	60.97%
Station 7	105	495	187	1.78	62.29%
Station 8	108	562	204	1.89	63.74%
Station 9	87	614	188	2.17	69.41%
Station 10	86	667	182	2.11	72.66%
Station 11	84	692	188	2.23	72.89%
Station 12	87	716	199	2.29	72.20%
Station 13	89	743	238	3.00	68.03%

reduces the overall delays of the trains at each station and thus improves the operation efficiency of the metro line system, especially for large-scale metro lines with a number of stations.

In addition, to demonstrate the reduced conservativeness of the stability condition of the Markovian model for describing the dynamic changing characteristic of the passengers arrival flow, we further conduct simulations for the case with the Markovian model and the case with the arbitrary switching model under the same initial conditions and disturbances. First, using the result in Proposition 3.1, we can obtain the stability condition of the metro line under the Markovian model, and the corresponding error state evolution trend for any station (here we choose station 13) is plotted as the solid line in Fig. 11. In addition, for the case with an arbitrary switching model of the passenger arrival flow, according to the stability results for the arbitrary switching model proposed by [8], the stability condition for the metro line under the arbitrary switching model can be derived. Under the stability condition, the corresponding error state evolution trend for station 13 under the arbitrary switching model is plotted as the dotted line in Fig. 11. By comparing the solid line and the dotted line in Fig. 11, we can find that the fluctuation of the error state for station 13 for the case with the Markovian model is obviously smaller than that for the case with the arbitrary switching model. This shows that the stability condition obtained from the Markovian model is less conservative than that obtained from the arbitrary switching model because the number of constraint conditions of linear matrix inequalities is increased under the arbitrary switching model.

4.3. Scenario 3: the stations are with different stochastic switching modes of passenger arrival rates

In Scenarios 1 and 2, only one of the realizations of discrete-time Markovian jumping modes is considered. In this part, we will design the robust train regulation under different stochastic switching modes of passenger arrival rates to show the stochastic stability of the metro line under the robust control. We also assume that train 10 is departing from the first station at 7:00 a.m.

For the stochastic passenger arrival flow, according to the probability transition matrix (43), a set of 15 different cases of the Markovian jumping modes are generated through Monte Carlo simulations, which are shown in Table 4. Then under the

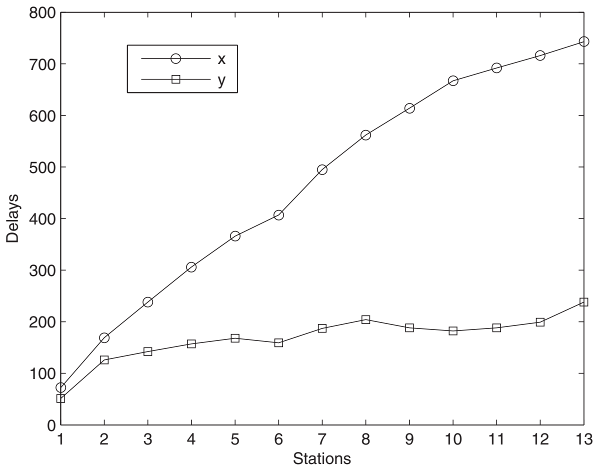

Fig. 10. The evolution trend of the overall delays of the trains at each station.

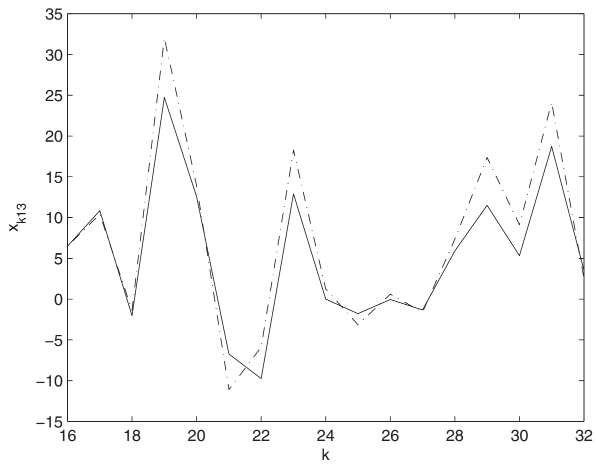

Fig. 11. The error state evolution trend for station 13 under stability conditions.

Table 4
The Markovian jumping modes with 15 cases.

Case 1	1 → 2 → 2 → 1 → 2 → 1 → 1 → 1 → 2 → 2 → 3 → 2 → 2 → 1 → 1 → 2 → 1
Case 2	1 → 1 → 1 → 3 → 2 → 1 → 1 → 2 → 3 → 1 → 1 → 1 → 1 → 1 → 1 → 1 → 1
Case 3	1 → 2 → 3 → 1 → 3 → 1 → 2 → 3 → 2 → 2 → 3 → 1 → 1 → 2 → 2 → 1 → 1
Case 4	1 → 2 → 1 → 2 → 2 → 2 → 1 → 2 → 2 → 2 → 1 → 2 → 1 → 1 → 3 → 2 → 2
Case 5	2 → 1 → 1 → 1 → 2 → 1 → 3 → 1 → 1 → 3 → 2 → 2 → 1 → 1 → 1 → 2 → 2
Case 6	1 → 1 → 1 → 1 → 1 → 2 → 1 → 2 → 2 → 2 → 2 → 2 → 3 → 1 → 2 → 1 → 1
Case 7	2 → 1 → 2 → 1 → 1 → 1 → 1 → 1 → 3 → 1 → 1 → 1 → 1 → 2 → 1 → 2 → 1
Case 8	1 → 1 → 1 → 2 → 3 → 1 → 1 → 1 → 2 → 3 → 2 → 3 → 3 → 1 → 3 → 3 → 2
Case 9	2 → 1 → 2 → 3 → 1 → 1 → 2 → 1 → 1 → 1 → 1 → 1 → 1 → 2 → 2 → 1 → 1
Case 10	2 → 1 → 2 → 1 → 2 → 3 → 3 → 1 → 1 → 2 → 2 → 1 → 1 → 1 → 1 → 1 → 1
Case 11	2 → 3 → 1 → 2 → 1 → 2 → 2 → 1 → 1 → 1 → 3 → 1 → 1 → 1 → 1 → 1 → 1
Case 12	1 → 2 → 1 → 1 → 2 → 1 → 3 → 1 → 1 → 1 → 1 → 2 → 3 → 2 → 3 → 2 → 2
Case 13	2 → 1 → 3 → 2 → 1 → 2 → 3 → 1 → 2 → 2 → 2 → 2 → 1 → 3 → 1 → 2 → 3
Case 14	1 → 1 → 1 → 1 → 2 → 1 → 1 → 1 → 2 → 1 → 2 → 3 → 2 → 1 → 1 → 1 → 1
Case 15	1 → 1 → 1 → 1 → 2 → 1 → 2 → 1 → 1 → 2 → 2 → 1 → 3 → 1 → 1 → 1 → 1

robust train regulation strategy, by applying Algorithm 3.1 to these different cases, Fig. 12 depicts the error state evolutions of train 10 at stations 5, 8, 10 and 13 with 15 different cases, respectively. The figure shows that the error state evolutions for 15 different cases are all in close proximity to each other for the four considered stations for train 10, and thus, the robust train regulation ensures the stochastic stability of a metro line system with stochastic passenger arrival flow.

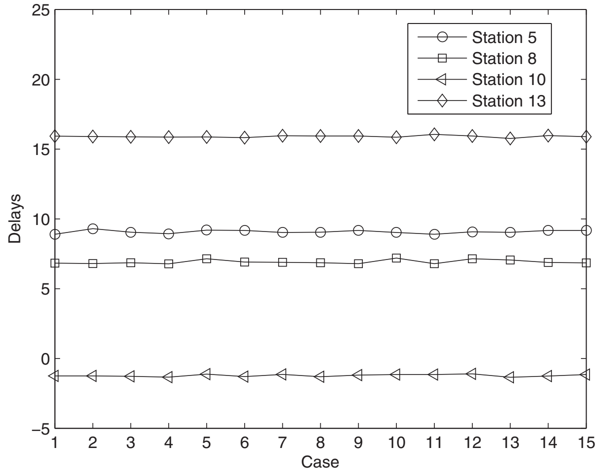

Fig. 12. The delays of the affected train 10 in 15 different cases for stations 5, 8, 10 and 13.

Table 5
The overall train disturbances and delays in 15 different cases.

	$\bar{w}$	$\bar{x}$	$\bar{y}$	$\bar{y}/\bar{w}$	$(\bar{x}-\bar{y})/\bar{x}$
Case 1	1143	6044	2182	1.91	63.89%
Case 2	1143	6043	2192	1.91	63.73%
Case 3	1143	6048	2194	1.92	63.71%
Case 4	1143	6045	2180	1.91	63.95%
Case 5	1143	6046	2191	1.92	63.76%
Case 6	1143	6042	2182	1.91	63.89%
Case 7	1143	6044	2187	1.91	63.81%
Case 8	1143	6044	2203	1.93	63.55%
Case 9	1143	6047	2186	1.91	63.85%
Case 10	1143	6048	2190	1.92	63.79%
Case 11	1143	6048	2190	1.92	63.79%
Case 12	1143	6045	2192	1.92	63.74%
Case 13	1143	6050	2190	1.92	63.80%
Case 14	1143	6041	2188	1.91	63.81%
Case 15	1143	6042	2188	1.91	63.79%

Moreover, let $\bar{w}$, $\bar{x}$, and $\bar{y}$ represent the overall disturbances, the overall delays without robust train regulation, and the overall delays with robust train regulation at all stations, respectively. By performing Algorithm 3.1 for 15 different cases, the corresponding overall disturbances and delays at all stations for the different cases are calculated and summarized in Table 5. From Table 5, we can observe that under the same disturbances, the proportion of the controlled delays to the input disturbances is effectively controlled to the range of 1.91 to 1.93 for all cases (see the fifth column of Table 5). In addition, the overall delays of all stations for all cases are reduced to the interval [63.71%, 63.95%] compared to the case without regulation. The corresponding overall delays of the trains in different cases are plotted in Fig. 13, which reveals that the robust train regulation strategy significantly reduces the overall delays for the different cases. In addition, the reduced overall delays of all stations for the different cases with different Markovian jumping modes are approximately the same, which indicates the stability and reliability of the proposed train regulation methods for the different Markovian jumping modes. In summary, the proposed robust train regulation strategy ensures the robustness and stability of metro line systems for recovering larger train delays. It should be noted that once the real-world time-dependent demand patterns and actual schedules/timetables for the metro line are obtained, the proposed methods can be easily applied to practical train regulations.

5. Conclusion

In this paper, the robust train regulation problem for metro lines with stochastic passenger arrival flow and uncertain disturbances is investigated. The passenger arrival flow is assumed to be dependent on a discrete Markovian process, and the dwell time of the train is characterized by dynamically stochastic switching at the different stages of the stations with a discrete Markovian jumping parameters. Then, a constrained state-space model for the train traffic of a metro line operation is developed based on the discrete Markovian system. Using stochastic stability theory, a sufficient condition for the

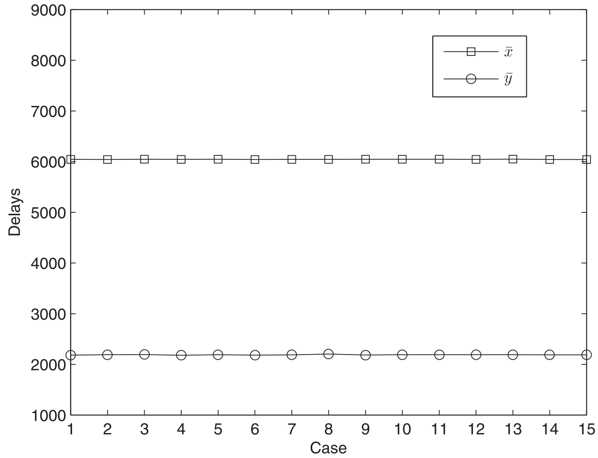

Fig. 13. The overall delays of the trains in 15 different cases.

existence of state-feedback control as the train regulation strategy is given in terms of linear matrix inequalities to ensure the stochastic stability of the train traffic. Moreover, the robust train regulation is designed to guarantee that the practical train timetable tracks the nominal timetable with a disturbance attenuation level. To obtain a smaller disturbance attenuation level, an effective iteration algorithm is proposed to solve a nonlinear optimization problem to determine the optimal state feedback control as the train regulation strategy. Numerical examples show that, under the proposed robust train regulation, the train delays are significantly reduced, and the operation efficiency of the metro line system is improved, especially for large-scale metro lines with a number of stations. Moreover, a number of Markovian jumping modes are generated via Monte Carlo simulations to demonstrate the stochastic stability of the metro line under the robust control. The main goal of the proposed regulation is a full timetable recovery. Thus, the proposed method is applicable to train delays and disturbances in a certain range. With great delays and disturbances, a new reference timetable should be designed, which is related to the rescheduling problem and the robust train rescheduling strategy needs to be investigated in the future. Additionally, the frequency and regularity of the train regulation problem for a metro line system will be another future research topic.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Nos. 71401007, 71422002, 71621001), and the Research Foundation of State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University (Nos. RCS2016ZJ002, RCS2016ZZ001).

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