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Energy consumption and travel time analysis for metro lines with express/local mode

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ABSTRACT

In recent years, some innovations have appeared in the operation of metro system to save energy consumption and speed up trains. Compared with the standard stop mode, in which a train stops at every station, express/local stop mode can lead to lower energy consumption and less travel time. This paper aims to find the relationship among energy consumption, travel time and timetables, and then obtain a more optimized solution via adjusting timetables. After analyzing the characteristics of express/local stop mode, we linearly formulate energy consumption and passenger travel time, and propose a bi-objective programming model to better understand the relationship between lowering energy consumption and reducing travel time. Taking Beijing Metro Line 6 as a numerical example, we compare the express/local mode and standard stop mode both in total travel time and energy consumption, illustrating the applicability of express/local mode.

1. Introduction

Due to its large-capacity and high-reliability, metro system has been adopted as a main type of rapid transit in many big cities. In the past decades, as the increase of environmental pressure, some important innovations have appeared in the operation of metro system to lower energy consumption, with little or no reduction on service quality, such as travel time. One of these innovation is designing more applicable stop modes, which may give rise to reduction both in energy consumption and travel time.

According to the characteristics of passenger flow, many metro systems have developed special stop modes, in which some trains need not stop at every station. Generally, there are three basic types of special stop modes (Vuchic, 2005), namely, *skip-stop*,*zonal* and *express/local*. For the first two operation modes, readers can refer to Vuchic (1973, 2005). In this paper, we focus on metro lines with express/local mode, which is the only way to provide regular service at all stations as well as higher-speed service at major stations. Express/local mode is adopted in some big cities with a large number of suburban commuters, such as Paris, Tokyo and New York, and sooner, it will be used in Beijing and Shanghai.

Since trains stop less in the express/local mode, which means that there are less accelerations and decelerations, energy consumption of train traction is reduced compared to that in the standard stop mode. However, the calculation of energy consumption is much more difficult if the express/local mode is used. The first reason is that, due to the introduction of express/local mode, passengers have more routes to choose, making it difficult to calculate passenger number in trains. As is known, the distribution of passengers affects traction energy consumption. The second reason is that, interaction

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between trains is much greater in the express/local mode. Note that in the express/local mode, an express train may overtake a local train at stations which are equipped with overtaking facilities. In order to ensure that overtaking happens at the right time and stations, timetables of express trains and local trains are highly interrelated, which complicates the calculation of energy consumption.

In this paper, we aim to reveal the relationship among energy consumption, travel time and timetables under the condition of express/local stop mode, and then propose a better solution, which leads to reduction both on energy consumption and travel time. Based on the analysis of route choice and energy consumption, we formulate linear functions for energy consumption and travel time in different OD (origin-destination) cases. Then, we build a bi-objective programming model to further understand the relationship among energy consumption, travel time and timetables. Specifically, the decision variables are timetables of express and local trains; the constraints set of the model, which mainly ensures the safety of operation, consists of link running time constraints, station dwelling constraints, and headway constraints. Taking Beijing Metro Line 6 as a numerical example, we compare the express/local mode and standard stop mode both in energy consumption and travel time. Moreover, we find that there exists a Pareto frontier between energy consumption and travel time.

Noted that trains in metro system are driven by electricity, and the actual electricity (fuel) consumption is roughly in proportion to energy consumption. For this reason, we use energy consumption to represent electricity consumption in this paper. The rest of this paper is organized as follows. Section 2 reviews relevant literature in recent years. In Section 3, the background of express/local stop mode is introduced, and the assumptions used in this paper are summarized. In Section 4, we formulate the model for energy consumption and travel time in detail. In Section 5, some numerical experiments are implemented to show the effectiveness of the proposed model and methods. Finally, a conclusion is made in Section 6.

2. Literature review

This paper aims to gain a better solution on energy consumption and travel time for a metro line with express/local mode via adjusting train timetable, so we first review some relevant literature on energy consumption of metro system. As pointed out by Tolliver et al. (2013) and Wang et al. (2014), rail is low energy-consumption in transiting per unit of passenger or freight, compared with road and air transportation. However, as environmental pressure is increasing in recent years, fuel conservation has become a hot issue in rail transport. Roughly speaking, fuel conservation methods includes energy-efficient operation and energy-efficient scheduling/timetable. The former applies the optimal control theory to optimize the speed profile between successive stations to minimize traction energy consumption, and relevant literature includes Howlett and Pudney (1995), Albrecht (2008) and Mensing et al. (2013). The latter aims to lower traction energy consumption via optimizing scheduling plans or timetables of trains, which is actually what this paper will do. In recent years, more and more researchers have used the latter methods in metro systems. Yang et al. (2012) made a preliminary theoretical discussion about the utilization of recovery energy, and presented a cooperative scheduling model for timetable optimization in subway systems. They first proposed cooperative scheduling rules and defined the overlapping time, and then an integer programming model was formulated to maximize the overlapping time, which was solved by a designed genetic algorithm. González-Gil et al. (2014) proposed an holistic approach to reduce the overall energy consumption of urban rail. The authors gave an insightful overview of energy usage in urban rail systems, and proposed a methodology to help implement energy saving schemes. Li and Lo (2014a) formulated an integrated energy-efficient model, jointly optimizing the timetable and speed profile as well. They further used genetic algorithm to solve the model and present some numerical experiments on Beijing Metro Yizhuang Line, the results of which showed significant improvement in energy consumption. Integrating the fluctuant passenger flow of metro line, Li and Lo (2014b) proposed a dynamic train scheduling and control framework to save energy. They first forecast the passenger demand, and then optimized timetables for the next cycle of the metro line. Moreover, they also optimized the speed profile of trains to reduce traction energy consumption and increase the storage of regenerative energy.

Train timetable problem always involves travel time, which is a significant indicator of service level. What is more, lowering energy consumption and reducing travel time is conflicting to some extend. As a result, some researchers simultaneously consider these two objectives in a model. Ghoseiri et al. (2004) developed a multi-objective programming model for the passenger train timetable problem, trying to lower fuel consumption cost and shorten passenger travel time simultaneously. To solve the model, the Pareto frontier was first determined via the $\epsilon-$constraint method, and then detailed multi-objective optimization was performed via the distance-based methods. Chevrier et al. (2013) proposed an approach to compute train running times by concurrently minimizing both energy consumption and running time. They gave a detailed analysis of speed profiling, and provided a set of tradeoff-solutions for decision-makers with the help of evolutionary algorithms. Sun et al. (2014) simultaneously investigated average travel time and energy consumption for high-speed trains, and an improved GA was designed to solve the problem. Yang et al. (2015) proposed an optimization method to schedule trains to simultaneously reduce energy consumption and travel time. An integer programming model with timetable and speed control was formulated. Then, they designed an optimal train control algorithm and an adaptive genetic algorithm to solve the model.

To our best knowledge, there is no analysis of energy consumption and travel time for metro lines with express/local mode in existing literature. Compared with that in standard stop mode, there are fewer stops and therefore fewer acceleration and deceleration phases in express/local mode, which indicates that energy consumption and travel time may be effectively lowered at the same time under some conditions. However, the express/local mode brings more difficulties in analysis and calculation. These all motivate us to investigate the energy consumption and travel time in the condition of express/local mode.

3. Problem description

In this section, we first describe the express/local stop mode in metro lines, and then present assumptions that this paper follows.

In urban metro lines, operating the express/local mode is the only way to provide regular service among all stations as well as higher-speed service at major stations. According to the difference of rail facilities, express/local modes can be roughly categorized into four types: (I) 2-track lines with 2-track stations, (II) 2-track lines with 4-track stations, (III) 3-track lines and (IV) 4-track lines (Vuchic, 2005). Among these four types, only type II involves overtaking, that is, express trains overtake local trains at some stations.

Fig. 1 shows a sketch map of a metro line with express/local mode. In Fig. 1, major stations are marked by solid dot, minor stations are marked by hollow dot, and the overtaking stations, which are chosen from major stations, are additionally marked by triangles. In Fig. 1, minor stations are $\{3, 5, 7, 9, 11\}$, major stations are $\{1, 2, 4, 6, 8, 10, 12\}$, and overtaking stations are $\{6, 10\}$. Local trains stop at all the stations along the line, while express trains stop and only stop at major stations. Note that overtaking is allowed, which only happens at the predetermined overtaking stations.

Fig. 2 illustrates the track and station design in express/local mode. The overtaking stations, equipped with four track lines, are labeled as "XT", the non-overtaking major stations are labeled as "X", and minor stations are labeled as "L". In the operation, a local train first enters an overtaking station, and dwells at the outer track; a certain time later, an express train enters the overtaking station, and dwells at the inner track. During the dwelling time, passengers can transfer from one train to the other. The express train departs from the overtaking station, and after a certain time, the local train departs, which implies that an overtaking is completed. At major stations, local trains serve as collectors-distributors for express trains. The time-distance diagram for this express/local mode is presented in Fig. 3, in which overtaking stations are station 6 and station 10.

Vuchic (2005) pointed out that the express/local mode performs well when (i) passengers' OD demand shows great heterogeneity, and (ii) the passenger volume and service frequency are both moderate. Due to the latter, we assume that passengers can always get on the first train that stops at their stations after their arrival. In other words, the over-crowded situation is not considered in this paper. Further, following assumptions hold through this paper:

(1) Passenger flow is steady during the time horizon, and the arrival time of passengers at each station is uniformly distributed.
(2) Local trains and express trains alternately depart, that is, the express/local ratio is 1:1.
(3) During the time horizon, timetables of local trains are parallel with fixed time interval; for timetables of express trains, it is the same.
(4) Local trains stop at every stations, while express trains stop and only stop at major stations.
(5) Express trains overtake local trains only at overtaking stations.
(6) Passengers can choose each type of trains freely, once they enter the metro system.
(7) All the trains have the same kinematical characteristics, *i.e.*, power, mass and resistance coefficients.

4. Formulation of programming model

In this section, we formulate a bi-objective programming model to quantitatively analyze the relationship among energy consumption, travel time and timetables when the express/local mode is used. Specifically, the decision variable of the model is timetable, and the objectives are train traction energy consumption and passengers' total travel time.

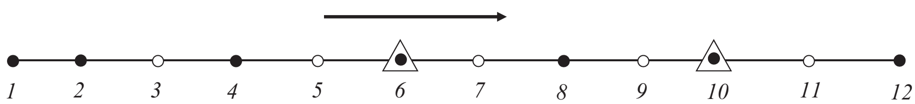

Fig. 1. A typical metro line with express/local mode.

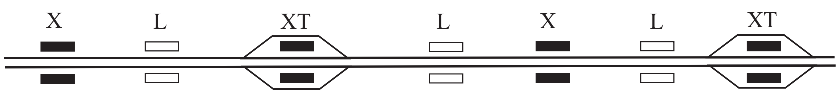

Fig. 2. Line and stations design for express/local modes.

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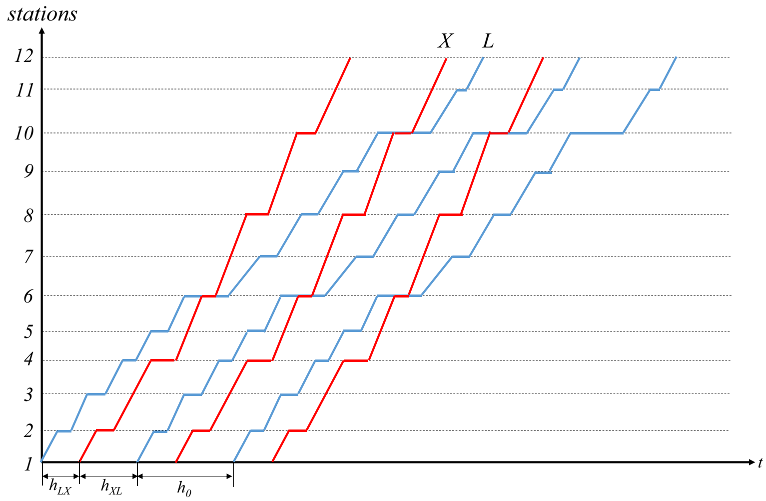

Fig. 3. The time-distance diagram for metro lines with express/local mode.

Table 1
List of parameters.

Notation	Definition
H	Time horizon under consideration
$\mathbf{N}$	Set of stations, $\mathbf{N} = \{1, 2, \ldots, n\}$, where 1 is originating station and n is terminal station
m	Number of major stations
$\mathbf{X}$	Set of major stations, $\mathbf{X} = \{X_1, \ldots, X_j, \ldots, X_m\} \subset \mathbf{N}$, where $X_1 = 1$ and $X_m = n$
$\tilde{m}$	Number of overtaking stations
$\tilde{\mathbf{X}}$	Set of overtaking stations, $\tilde{\mathbf{X}} = \{\tilde{X}_1, \ldots, \tilde{X}_{\tilde{m}}\} \subset \mathbf{X}$
$len(i)$	Length of the railway link i, namely, from station i to station $i+1, i = 1, 2, \ldots, n-1$
t^L_{1i}, t^L_{2i}	Minimum and maximum running time of local trains on railway link i
t^X_{1j}, t^X_{2j}	Minimum and maximum running time of express trains from major station X_j to major station $X_{j+1}, j = 1, 2, \ldots, m-1$
h_0	Departure interval between two successive local(express) trains at the originating station
λ_{pq}	Number of passengers from station p to station q during every h_0
h_s	Minimum departure interval between successive trains at the originating station
h_w	Minimum headway between successive trains on the same link
h_r	Minimum interval between a train's departure and the behind train's arrival at a station
d_0	Minimum dwelling time at a station
a	Maximum acceleration of the train
b	Maximum braking of the train
c	Resistance force on per unit of the train, or deceleration generated by resistance force
P	Average mass of each passenger
G	Mass of a train

The traction energy is consumed to overcome rolling resistance, the aerodynamic resistance and the acceleration force during the train's movement from one station to another. The optimal speed profile of trains has been an important topic for years (Howlett and Pudney, 1995; Albrecht, 2008). In nowadays, when the departure time and arrival time are given, metro trains can usually follow a speed profile which leads to the minimum traction energy consumption. For this reason, in this paper, energy consumption that we want to calculate is the minimum energy consumption when the timetable is given.

Before formulating the model, we first summarize the notations that will be used throughout the paper in Table 1.

4.1. Decision variables

As mentioned before, this paper aims to optimize timetables of express trains and local trains. In assumption (3), timetables of the same type of trains are parallel, so we only need to optimize timetables of the first pair of express/local trains, and the departure time interval between them.

Since energy consumption has a direct relationship with running time, we choose link running time, station dwelling time and departure time interval between express train and local train as decision variables, which are listed and explained as follows. Note that timetables of express/local trains are actually composed by these decision variables.

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t_i^L	Running time of local trains on the railway link i, $i = 1, 2, \ldots, n-1$
t_j^X	Running time of express trains from major station X_j to major station $X_{j+1}, j = 1, 2, \ldots, m-1$
d_i^L	Dwelling time of local trains at station $i, i = 2, \ldots, n-1$
d_j^X	Dwelling time of express trains at major station $X_j, j = 2, \ldots, m-1$
h_{LX}	Departure interval between local train and express train at the originating station
h_{XL}	Departure interval between express train and local train at the originating station

The link running time is usually estimated by adding a time supplement on the shortest running time. Via adjusting link running time, station dwelling times and departure intervals, the safety and efficiency of metro lines can be improved.

4.2. Formulation of the constraints

For metro lines, train timetable problems should satisfy three main types of constraints, namely, link running time constraints, station dwelling time constraints and headway constraints. In this part, we will formulate these constraints in the context of express/local mode in detail.

4.2.1. Link running time constraints

In common situations, for a local train running on link i, link running time t_i^L should not be greater than t_{2i}^L, or less than t_{1i}^L. The bounds t_{1i}^L and t_{2i}^L are determined by rail facilities and management requirements. Thus, link running time constraints for local trains are

$$t_{2i}^L \geqslant t_i^L \geqslant t_{1i}^L, \quad i = 1, 2, \ldots, n-1. \tag{1}$$

Similarly, for express trains running from major station X_j to major station X_{j+1}, we have the following constraints

$$t_{2j}^X \geqslant t_j^X \geqslant t_{1j}^X, \quad j = 1, 2, \ldots, m-1. \tag{2}$$

4.2.2. Station dwelling time constraints

To ensure the quality of metro service, dwelling time at each station should not be less than d_0, *i.e.*,

$$d_i^L \geqslant d_0, \quad i = 2, 3, \ldots, n-1 \tag{3}$$

$$d_j^X \geqslant d_0, \quad j = 2, 3, \ldots, m-1. \tag{4}$$

As the design of the express/local mode, dwelling time of local trains should cover that of express trains at the overtaking stations, *i.e.*,

$$d_{X_j}^L > d_j^X, \quad X_j \in \tilde{\mathbf{X}}.$$

Later, we will find that the above constraints are self-evident if headway constraints are satisfied, which indicates that they will not be explicitly involved in the model. Besides, dwelling time of local trains at overtaking stations should not be too long, otherwise passengers in local trains will complain. As a result, at overtaking stations, the following constraints must be satisfied

$$d_{X_j}^L \leqslant d_1, \quad j = 2, 3, \ldots, m-1, \tag{5}$$

where d_1 is the upper bound of dwelling time.

4.2.3. Headway constraints

Headway constraints ensure that no collision of trains occurs in metro lines. Fig. 4 presents a time-distance diagram for the metro line with express/local mode, in which some important time intervals are marked. At the originating station, h_{LX}, time interval between departure of a local train and that of a successive express train, should not be less than the minimum time interval h_s, *i.e.*,

$$h_{LX} \geqslant h_s. \tag{6}$$

Similarly, h_{XL} should satisfy

$$h_{XL} \geqslant h_s. \tag{7}$$

According to the definition of h_0, the following equation always holds

$$h_{LX} + h_{XL} = h_0. \tag{8}$$

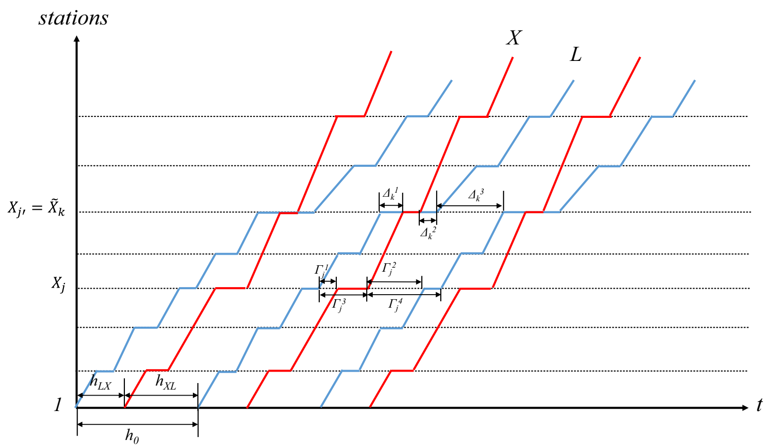

Fig. 4. Illustration for headway constraints.

Note that in this paper, h_0 is assumed to be predetermined.

At major stations where overtaking happens, for instance $X_{j'} = \tilde{X}_k \in \tilde{\mathbf{X}}$ (see Fig. 4), a local train arrives first, and after a time of Δ_k^1, an express train arrives. To reduce the risk of collision, time interval Δ_k^1 is required to be greater than h_w, *i.e.*,

$$\Delta_k^1 = \left(h_{LX} + (k-1)h_0 + \sum_{1 \leqslant u \leqslant j'-1} t_u^X + \sum_{2 \leqslant u \leqslant j'-1} d_u^X \right) - \left(\sum_{1 \leqslant v \leqslant X_{j'}-1} t_v^L + \sum_{2 \leqslant v \leqslant X_{j'}-1} d_v^L \right) \geqslant h_w, \quad k = 1, \ldots, \tilde{m}. \tag{9}$$

After a certain dwelling time, the express train departs from overtaking station $X_{j'}$, followed by a local train. The departure time interval Δ_k^2 is required to be greater than h_r, *i.e.*,

$$\Delta_k^2 = \left(\sum_{1 \leqslant v \leqslant X_{j'}-1} t_v^L + \sum_{1 \leqslant v \leqslant X_{j'}} d_v^L \right) - \left(h_{LX} + (k-1)h_0 + \sum_{1 \leqslant u \leqslant j'-1} t_u^X + \sum_{2 \leqslant u \leqslant j'} d_u^X \right) \geqslant h_w, \quad k = 1, \ldots, \tilde{m}. \tag{10}$$

At major stations, constraints (9) and (10) indicate that

$$d_{\tilde{X}_k}^L - d_{j'}^X \geqslant h_w + h_r > 0, \quad \text{for } X_{j'} = \tilde{X}_k, k = 1, \ldots, \tilde{m}.$$

which leads to the requirement that the dwelling time of local trains should cover that of express trains at major stations. However, the local train can not stop at major stations too long to impede the arrival of next local train. Considering the minimum headway h_w, the following constraints are required,

$$\Delta_k^3 = h_0 - d_{\tilde{X}_k}^L \geqslant h_r, \quad \text{for } X_{j'} = \tilde{X}_k, k = 1, \ldots, \tilde{m}. \tag{11}$$

For other major stations, such as station X_j in Fig. 4, the similar headway constraints exist, *i.e.*,

$$\Gamma_j^1 \geqslant h_r, \quad \Gamma_j^2 \geqslant h_r, \quad \text{for } X_j \in \mathbf{X} \setminus \tilde{\mathbf{X}}, \tag{12}$$

$$\Gamma_j^3 \geqslant h_w, \quad \Gamma_j^4 \geqslant h_w, \quad \text{for } X_j \in \mathbf{X} \setminus \tilde{\mathbf{X}}. \tag{13}$$

4.3. Objective functions

With the introduction of express/local mode, passengers from station p to station q may have more than one routes. As a result, it's difficult to obtain the number of passengers in train, which is a key factor in calculating traction energy consumption and total travel time. In Appendix A, we analyze the route choice in express/local mode, and propose a method to estimate passengers number. Simply, assume that p is a local station and p_X is the first overtaking station after p. If transfer to an express train at p_X can reduce travel time, the percent of passengers that will transfer is K_{pq}, where K_{pq} is defined by

$$K_{pq} = (q-p)/n, \quad 1 \leqslant p < q \leqslant n.$$

In other words, the transfer percent K_{pq} is a linear function of the locations of origin station p and destination station q.

In Appendix B, we deduce a linear function to approximately describe the relationship between energy consumption and link running time. Based on the above work, we then formulate the objective functions of energy consumption and travel time, which is the core part of this paper. Due to the diversified routes, the formulation has to break down into different cases, depending on origin-destination of travels.

Since the timetable is cyclical and passenger flow is assumed steady, we only need to analyze the travel time and energy consumption during time h_0. The aforementioned method for obtaining passengers number is based on route travel time, so for simplicity, total travel time is chosen as the first objective, and energy consumption the second.

Travel time consists of running time on links and dwelling times at stations. To formulate total travel time, we are required to sum all the passengers' running time in trains and dwelling times at stations. For the sake of convenience, the set of stations is segmented by overtaking stations. Take the metro line in Fig. 5 as an example, in which $S_1 = \{1,2,3,4,5\}, S_2 = \{6,7,8,9\}$ and $S_3 = \{10,11,12\}$. Obviously, if the number of overtaking stations is $\tilde{m}$, then there are $\tilde{m}+1$ segments.

Without loss of generality, assume that station p is the starting point and station q is the end point of the travel, $p < q$. If a passenger takes a local train to station q without transferring, the travel time without waiting time at station p is

$$T^L(p,q) = \sum_{p \leqslant v \leqslant q-1} t_v^L + \sum_{p+1 \leqslant v \leqslant q-1} d_v^L. \tag{14}$$

The first part of $T^L(p,q)$ is the sum of running time, and the second part is the sum of dwelling time at intermediate stations. Note that $T^L(p,q)$ is a basic function for the calculation of travel time.

We define a function $x(\cdot)$ from set $\mathbf{X}$ to $\{1,2,\ldots,m\}$, where $\mathbf{X}$ is the set of major stations and m is the number of major stations, *i.e.*,

$$x(X_j) = j, \quad X_j \in \mathbf{X}.$$

Simply speaking, $x(\cdot)$ is the index function of major stations.

When $p,q \in \mathbf{X}$, if a passenger takes an express train to station q, the travel time without waiting time at station p is

$$T^X(p,q) = \sum_{x(p) \leqslant u \leqslant x(q)-1} t_u^X + \sum_{x(p)+1 \leqslant v \leqslant x(q)-1} d_u^X. \tag{15}$$

Note that $T^X(p,q)$ is a basic function for calculating travel time. The similar functions can be defined for the calculation of energy consumption, that is, moving per unit mass from station p to station q by local trains and express trains, the traction energy consumption can be respectively written as

$$W^L(p,q) = \sum_{p \leqslant v \leqslant q-1} E_v^L(t_v^L), \tag{16}$$

$$W^X(p,q) = \sum_{x(p) \leqslant u \leqslant x(q)-1} E_u^X(t_u^X). \tag{17}$$

In function (16), $E_v^L(t_v^L)$ represents local train's minimum energy consumption on link v during running time t_v^L, and the same applies to $E_u^X(t_u^X)$ in function (17). For calculating $E_v^L(t_v^L)$ and $E_u^X(t_u^X)$, we use Algorithm 1 in Appendix B.

With the help of basic functions (14)–(17), we only need to pay attention to passenger' route choice when formulating total travel time and traction energy consumption. First at all, traction energy consumption in moving an empty local train and an empty express train from station 1 to station n can be formulated by following expression

$$W_{train} = G(W^L(1,n) + W^X(1,n)), \tag{18}$$

where G is the mass of a train.

Next, the analysis for total travel time and traction energy consumption breaks down into the following cases. Still, p is the starting point and station q is the end point.

Case 1: $p \in S_k$ and $q \in S_k$, which means that passengers travel within the same segment, for example, from station 2 to station 5, or from station 6 to station 8. According to station types, this case further breaks down into 4 subcases.

Case 1a: q is a minor station, *i.e.*, $q \notin \mathbf{X}$. In this case, from station p to station q, taking a local train is almost the best choice, no matter station p is a minor station or a major station. Thus, the travel time is

$$\frac{h_0}{2} + T^L(p,q),$$

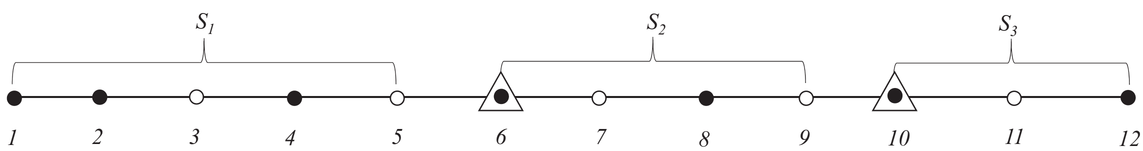

Fig. 5. Segment of stations.

where the first part is the average waiting time at station i, the second part is obtained by function (14).

Considering the number of passengers is λ_{ij} during h_0, the total travel time for this part of passengers is

$$T^{(1a)} = \sum_{1 \leqslant k \leqslant \tilde{m}+1} \sum_{\substack{p,q \in S_k \\ q \notin \mathbf{X}}} \lambda_{pq} \left(\frac{h_0}{2} + T^L(p,q) \right). \tag{19}$$

Similarly, the energy consumption for moving these passengers is

$$W^{(1a)}_{png} = \sum_{1 \leqslant k \leqslant \tilde{m}+1} \sum_{\substack{p,q \in S_k \\ q \notin \mathbf{X}}} \lambda_{pq} PW^L(p,q). \tag{20}$$

Case 1b: q is a major station, and p is a minor station, *i.e.*, $q \in \mathbf{X}$ and $p \notin \mathbf{X}$. In this case, passengers will take a local train, which means that the formulations of travel time and energy consumption are similar to those of Case 1a, *i.e.*,

$$T^{(1b)} = \sum_{1 \leqslant k \leqslant \tilde{m}+1} \sum_{\substack{p,q \in S_k \\ p \notin \mathbf{X}, q \in \mathbf{X}}} \lambda_{pq} \left(\frac{h_0}{2} + T^L(p,q) \right), \tag{21}$$

$$W^{(1b)}_{png} = \sum_{1 \leqslant k \leqslant \tilde{m}+1} \sum_{\substack{p,q \in S_k \\ p \notin \mathbf{X}, q \in \mathbf{X}}} \lambda_{pq} PW^L(p,q). \tag{22}$$

Case 1c: both p and q are major stations, and p is not an overtaking station, *i.e.*, $q \in \mathbf{X}$ and $p \in \mathbf{X} \setminus \tilde{\mathbf{X}}$. This case is illustrated in Fig. 6, *i.e.*, the travel is from station p_{1c} to q_{1c}.

In this case, passengers in time interval $[t_1, t_2]$ will take an express train and passengers in time interval $[t_2, t_3]$ will take a local train. In average, the numbers of these two parts of passengers are $\frac{t_2 - t_1}{h_0} \lambda_{pq}$ and $\frac{t_3 - t_2}{h_0} \lambda_{pq}$, respectively. Note that $t_3 - t_1 = h_0$ and t_2 is determined by the departure interval h_{LX}, link running time and station dwelling time. For simplicity, we assume that $(t_2 - t_1) = (t_3 - t_2)$, which means that the numbers of passengers taking an express train and a local train are both about $\frac{1}{2}\lambda_{pq}$. Actually, in metro lines with about 3 *mins* departure interval, there is little difference between $(t_2 - t_1)$ and $(t_3 - t_2)$. As a result, we have the following formulations for travel time and energy consumption

$$T^{(1c)} = \sum_{1 \leqslant k \leqslant \tilde{m}+1} \sum_{\substack{p,q \in S_k \\ p \in \mathbf{X} \setminus \tilde{\mathbf{X}}, q \in \mathbf{X}}} \frac{1}{2} \lambda_{pq} \left(\left(\frac{h_0}{4} + T^X(p,q) \right) + \left(\frac{h_0}{4} + T^L(p,q) \right) \right), \tag{23}$$

and

$$W^{(1c)}_{png} = \sum_{1 \leqslant k \leqslant \tilde{m}+1} \sum_{\substack{p,q \in S_k \\ p \in \mathbf{X} \setminus \tilde{\mathbf{X}}, q \in \mathbf{X}}} \frac{1}{2} \lambda_{pq} P \left(W^X(p,q) + W^L(p,q) \right). \tag{24}$$

Case 1d: both p and q are major stations, and p is an overtaking station, *i.e.*, $q \in \mathbf{X}$ and $p \in \tilde{\mathbf{X}}$. This case is illustrated in Fig. 6, *i.e.*, the travel is from station p_{1d} to station q_{1d}. Note that $t_5 - t_4 = h_0$.

As is discussed in Appendix A, at overtaking station p, about K_{pq} percent of passengers will take an express train. As a result, we have the following formulations

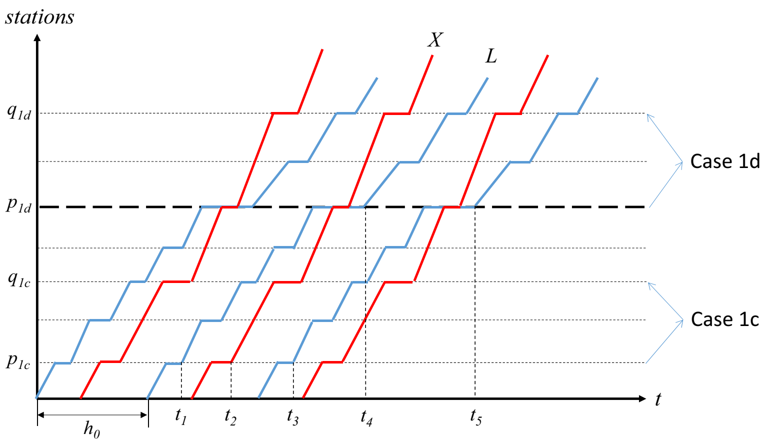

Fig. 6. Illustration for Case 1c and Case 1d.

$$T^{(1d)} = \sum_{1\leqslant k\leqslant \tilde{m}+1} \sum_{\substack{p,q\in S_k \\ p\in\tilde{\mathbf{X}},q\in\mathbf{X}}} \lambda_{pq}\left(K_{pq}\left(\frac{h_0}{2}+T^X(p,q)\right)+(1-K_{pq})\left(\frac{h_0}{2}+T^L(p,q)\right)\right), \tag{25}$$

and

$$W^{(1d)}_{png} = \sum_{1\leqslant k\leqslant \tilde{m}+1} \sum_{\substack{p,q\in S_k \\ p\in\tilde{\mathbf{X}},q\in\mathbf{X}}} \lambda_{pq}P\left(K_{pq}W^X(p,q)+(1-K_{pq})W^L(p,q)\right). \tag{26}$$

In short, travel time and energy consumption of Case 1 are

$$T^{(1)} = T^{(1a)} + T^{(1b)} + T^{(1c)} + T^{(1d)},$$

and

$$W^{(1)}_{png} = W^{(1a)}_{png} + W^{(1b)}_{png} + W^{(1c)}_{png} + W^{(1d)}_{png}.$$

Case 2: $p \in S_k$ and $q \in S_l, \tilde{m}+1 \geqslant l > k$, which means that passengers travel from one segment to another one, for example, from station 4 to station 7, or from station 3 to station 12. According to station types, this case further breaks down into 6 subcases.

Case 2a: both p and q are minor stations, *i.e.*, $p, q \notin \mathbf{X}$. This case is illustrated in Fig. 7, *i.e.*, the travel is from station p_{2a} to q_{2a}. In this case, passengers will first take a local train to the overtaking station $\tilde{X}_k$. At station $\tilde{X}_k$, about K_{pq} of them will transfer to an express train to go to the overtaking station $\tilde{X}_{l-1}$, and then transfer to a local train to arrival at station q. The rest passengers will not transfer and go to station q by the local train. Thus, travel time and energy consumption of this part passengers can be formulated respectively as follows

$$\begin{aligned} T^{(2a)} = \sum_{\substack{1\leqslant k\leqslant l \\ l\leqslant \tilde{m}+1}} \sum_{\substack{p\in S_k,q\in S_l \\ p,q\notin\mathbf{X}}} \lambda_{pq}\Bigg(\frac{h_0}{2} &+ T^L(p,\tilde{X}_k) + K_{pq}\left((\Delta^1_k + d^X_{x(\tilde{X}_k)}) + T^X(\tilde{X}_k,\tilde{X}_{l-1}) + (d^L_{\tilde{X}_{l-1}} - \Delta^1_{l-1}) + T^L(\tilde{X}_{l-1},q)\right) \\ &+(1-K_{pq})\left(d^L_{\tilde{X}_k} + T^L(\tilde{X}_k,q)\right)\Bigg), \end{aligned} \tag{27}$$

and

$$W^{(2a)}_{png} = \sum_{\substack{1\leqslant k< l \\ l\leqslant \tilde{m}+1}} \sum_{\substack{p\in S_k,q\in S_l \\ p,q\notin\mathbf{X}}} \lambda_{pq}P\left(W^L(p,\tilde{X}_k) + K_{pq}\left(W^X(\tilde{X}_k,\tilde{X}_{l-1}) + W^L(\tilde{X}_{l-1},q)\right) + (1-K_{pq})W^L(\tilde{X}_k,q)\right), \tag{28}$$

where Δ^1_k is defined in expression (9).

Case 2b: q is a minor station, and p is a non-overtaking major station, *i.e.*, $q \notin \mathbf{X}$ and $p \in \mathbf{X}\setminus\tilde{\mathbf{X}}$. This case is illustrated in Fig. 8, *i.e.*, the travel is from station p_{2b} to q_{2b}. As that in Case 1c, it is reasonable to assume that $t_2 - t_1 = t_3 - t_2 = \frac{h_0}{2}$.

At station p, passengers in time interval $[t_1, t_2]$ will first take an express train to the overtaking station $\tilde{X}_{l-1}$, then transfer to a local train, by which they will arrive at station q. Passengers in time interval $[t_2, t_3]$ will first take a local train to the overtaking station $\tilde{X}_k$, then K_{pq} of them will transfer to an express train to go to the overtaking station $\tilde{X}_{l-1}$, at which they will transfer to a local train to go to station q; the rest of them will take a local train to station q without transferring. Thus, travel time and energy consumption of this part passengers can be formulated respectively as follows

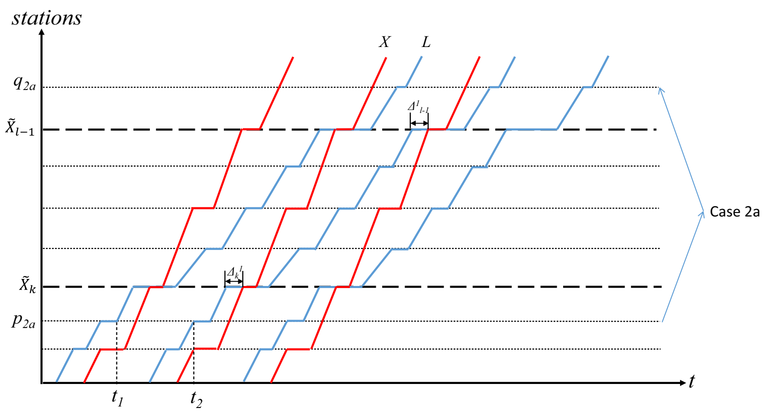

Fig. 7. Illustration for Case 2a.

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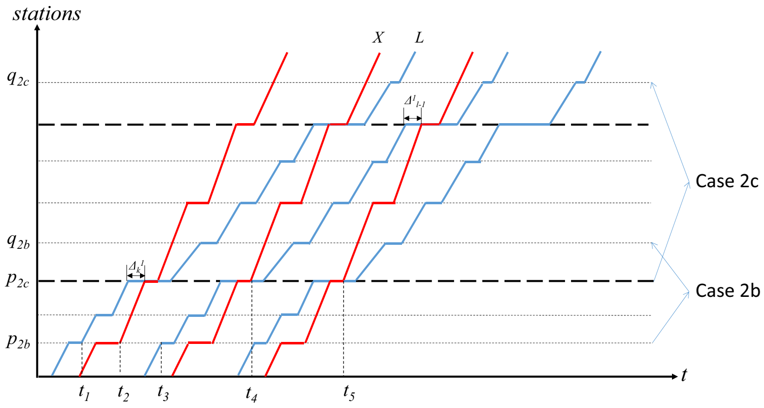

Fig. 8. Illustration for Cases 2b and Case 2c.

$$T^{(2b)} = \sum_{\substack{1\leqslant k<l\\ l\leqslant \bar{m}+1}} \sum_{\substack{p\in S_k, q\in S_l\\ p\in \mathbf{X}\setminus \tilde{\mathbf{X}}, q\notin \mathbf{X}}} \frac{1}{2}\lambda_{pq}\Bigg(\left(\frac{h_0}{4} + T^X(p,\tilde{X}_{l-1}) + (d^L_{\tilde{X}_{l-1}} - \Delta^1_{l-1}) + T^L(\tilde{X}_{l-1},q)\right) + K_{pq}\left(\frac{h_0}{4} + T^L(p,\tilde{X}_k) + (\Delta^1_k + d^X_{x(\tilde{X}_k)}) + T^X(\tilde{X}_k,\tilde{X}_{l-1}) + (d^L_{\tilde{X}_{l-1}} - \Delta^1_{l-1}) + T^L(\tilde{X}_{l-1},q)\right) + (1-K_{pq})\left(\frac{h_0}{4} + T^L(p,q)\right)\Bigg), \tag{29}$$

and

$$W^{(2b)}_{png} = \sum_{\substack{1\leqslant k<l\\ l\leqslant \bar{m}+1}} \sum_{\substack{p\in S_k, q\in S_l\\ p\in \mathbf{X}\setminus \tilde{\mathbf{X}}, q\notin \mathbf{X}}} \frac{1}{2}\lambda_{pq}P\Big(\left(W^X(p,\tilde{X}_{l-1}) + W^L(\tilde{X}_{l-1},q)\right) + K_{pq}\left(W^L(p,\tilde{X}_k) + W^X(\tilde{X}_k,\tilde{X}_{l-1}) + W^L(\tilde{X}_{l-1},q)\right) + (1-K_{pq})W^L(p,q)\Big). \tag{30}$$

Case 2c: q is a minor station, and p is an overtaking major station, *i.e.*, $q \notin \mathbf{X}$ and $p \in \tilde{\mathbf{X}}$. This case is illustrated in Fig. 8, *i.e.*, the travel is from station p_{2c} to q_{2c}. Note that $t_5 - t_4 = h_0$.

This case is similar to Case 1d, that is, at station p there are about $K_{pq}\lambda_{pq}$ passengers will take an express train to the last overtaking station before q, *i.e.*, $\tilde{X}_{l-1}$, then transfer to a local train, by which they will arrive at station q; the rest passengers will take a local train to station q directly. Travel time and energy consumption of this part passengers can be formulated respectively as follows

$$T^{(2c)} = \sum_{\substack{1\leqslant k<l\\ l\leqslant \bar{m}+1}} \sum_{\substack{p\in S_k, q\in S_l\\ p\in \tilde{\mathbf{X}}, q\notin \mathbf{X}}} \lambda_{pq}\left(K_{pq}\left(\frac{h_0}{2} + T^X(p,\tilde{X}_{l-1}) + (d^L_{\tilde{X}_{l-1}} - \Delta^1_{l-1}) + T^L(\tilde{X}_{l-1},q)\right) + (1-K_{pq})\left(\frac{h_0}{2} + T^L(p,q)\right)\right), \tag{31}$$

and

$$W^{(2c)}_{png} = \sum_{\substack{1\leqslant k<l\\ l\leqslant \bar{m}+1}} \sum_{\substack{p\in S_k, q\in S_l\\ p\in \tilde{\mathbf{X}}, q\notin \mathbf{X}}} \lambda_{pq}P\left(K_{pq}\left(W^X(p,\tilde{X}_{l-1}) + W^L(\tilde{X}_{l-1},q)\right) + (1-K_{pq})W^L(p,q)\right). \tag{32}$$

Case 2d: q is a major station, and p is a minor station, *i.e.*, $q \in \mathbf{X}$ and $p \notin \mathbf{X}$. This case is illustrated in Fig. 9, *i.e.*, the travel is from station p_{2d} to q_{2d}.

Since p is a minor station, passengers will first take a local train to the overtaking station, *i.e.*, $\tilde{X}_k$. At station $\tilde{X}_k$, about $K_{pq}\lambda_{pq}$ of them will transfer to an express train to station q, and the rest of them will take the local train to station q without transferring. Travel time and energy consumption of this part passengers can be formulated respectively as follows

$$T^{(2d)} = \sum_{\substack{1\leqslant k<l\\ l\leqslant \bar{m}+1}} \sum_{\substack{p\in S_k, q\in S_l\\ p\notin \mathbf{X}, q\in \mathbf{X}}} \lambda_{pq}\left(\left(\frac{h_0}{2} + T^L(p,\tilde{X}_k)\right) + K_{pq}\left((\Delta^1_k + d^X_{x(\tilde{X}_k)}) + T^X(\tilde{X}_k,q)\right) + (1-K_{pq})\left(d^L_{\tilde{X}_k} + T^L(\tilde{X}_k,q)\right)\right), \tag{33}$$

and

$$W^{(2d)}_{png} = \sum_{\substack{1\leqslant k<l\\ l\leqslant \bar{m}+1}} \sum_{\substack{p\in S_k, q\in S_l\\ p\notin \mathbf{X}, q\in \mathbf{X}}} \lambda_{pq}P\left(W^L(p,\tilde{X}_k) + K_{pq}W^X(\tilde{X}_k,q) + (1-K_{pq})W^L(\tilde{X}_k,q)\right). \tag{34}$$

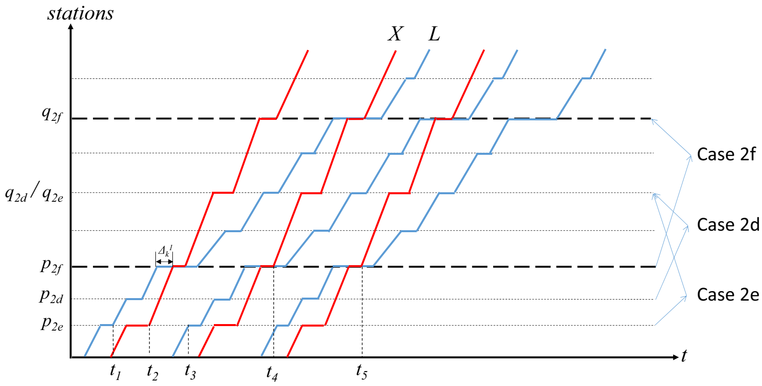

Fig. 9. Illustration for Case 2d, Case 2e and Case 2f.

Case 2e: both p and q are major stations, and p is a non-overtaking station, *i.e.*, $q \in \mathbf{X}$ and $p \in \mathbf{X} \setminus \tilde{\mathbf{X}}$. This case is illustrated in Fig. 9, *i.e.*, the travel is from station p_{2d} to q_{2d}. As that in Case 1c, it is reasonable to assume that $t_2 - t_1 = t_3 - t_2 = \frac{h_0}{2}$.

At station p, passengers in time interval $[t_1, t_2]$ will take an express train to station q with no transfer. Passengers in time interval $[t_2, t_3]$ will first take a local train to the overtaking station $\tilde{X}_k$, and at station $\tilde{X}_k$, about K_{pq} of them will transfer to an express train to go to station q; the rest of them will not transfer to an express train. Thus, travel time and energy consumption in Case 2e can be formulated respectively as follows

$$T^{(2e)} = \sum_{\substack{1 \leqslant k < l \\ l \leqslant \bar{m}+1}} \sum_{\substack{p \in S_k, q \in S_l \\ p \in \mathbf{X} \setminus \tilde{\mathbf{X}}, q \in \mathbf{X}}} \frac{1}{2} \lambda_{pq} \left(\left(\frac{h_0}{4} + T^X(p,q) \right) + K_{pq} \left(\frac{h_0}{4} + T^L(p, \tilde{X}_k) + (\Delta_k^1 + d^X_{x(\tilde{X}_k)}) + T^X(\tilde{X}_k, q) \right) + (1 - K_{pq}) \left(\frac{h_0}{4} + +T^L(p,q) \right) \right), \tag{35}$$

and

$$W^{(2e)}_{png} = \sum_{\substack{1 \leqslant k < l \\ l \leqslant \bar{m}+1}} \sum_{\substack{p \in S_k, q \in S_l \\ p \in \mathbf{X} \setminus \tilde{\mathbf{X}}, q \in \mathbf{X}}} \frac{1}{2} \lambda_{pq} P \left(W^X(p,q) + K_{pq} \left(W^L(p, \tilde{X}_k) + W^X(\tilde{X}_k, q) \right) + (1 - K_{pq}) W^L(p,q) \right). \tag{36}$$

Case 2f: both p and q are major stations, and p is an overtaking station, *i.e.*, $q \in \mathbf{X}$ and $p \in \tilde{\mathbf{X}}$. This case is illustrated in Fig. 9, *i.e.*, the travel is from station p_{2f} to q_{2f}.

At station p, about K_{pq} of passengers will take an express train to station q without transfer, and the rest will take a local train to station q without transfer. Thus, travel time and energy consumption in case2e can be formulated respectively as follows

$$T^{(2f)} = \sum_{\substack{1 \leqslant k < l \\ l \leqslant \bar{m}+1}} \sum_{\substack{p \in S_k, q \in S_l \\ p \in \tilde{\mathbf{X}}, q \in \mathbf{X}}} \lambda_{pq} \left(K_{pq} \left(\frac{h_0}{2} + T^X(p,q) \right) + (1 - K_{pq}) \left(\frac{h_0}{2} + T^L(p,q) \right) \right), \tag{37}$$

and

$$W^{(2f)}_{png} = \sum_{\substack{1 \leqslant k < l \\ l \leqslant \bar{m}+1}} \sum_{\substack{p \in S_k, q \in S_l \\ p \in \tilde{\mathbf{X}}, q \in \mathbf{X}}} \lambda_{pq} P \left(K_{pq} W^X(p,q) + (1 - K_{pq}) W^L(p,q) \right). \tag{38}$$

In short, travel time and energy consumption of Case 2 are

$$T^{(2)} = T^{(2a)} + T^{(2b)} + T^{(2c)} + T^{(2d)} + T^{(2e)} + T^{(2f)},$$

and

$$W^{(2)}_{png} = W^{(2a)}_{png} + W^{(2b)}_{png} + W^{(2c)}_{png} + W^{(2d)}_{png} + W^{(2e)}_{png} + W^{(2f)}_{png}.$$

Thus, passengers' total travel time and train traction energy consumption during h_0 is

$$T_{total} = T^{(1)} + T^{(2)}, \tag{39}$$

$$W_{total} = W_{train} + W^{(1)}_{png} + W^{(2)}_{png}. \tag{40}$$

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4.4. Construction of bi-objective programming model

Combining the objective functions (39) and (40) and constraints (1)–(11), we build the bi-objective programming model for metro lines with express/local mode:

$$\begin{cases} \min & (T_{total}, W_{total}) \\ s.t. & \text{constraints } (1)-(11). \end{cases} \tag{41}$$

Note that T_{total} and W_{total} are both linear functions of decision variables. Moreover, constraints (1)–(11) are all linear, so model (41) is a linear programming model. If we are only interested in one objective, model (41) degenerates to a single objective linear programming model, the optimal solution of which can be easily obtained by almost all the optimization softwares.

5. Numerical experiment

In this section, Beijing Metro Line 6 is taken as a numerical example to illustrate the effectiveness of the model. Our workstation is a personal computer with Intel(R) Core(TM) i5 3337U 1.80 GHz CPU and 4.00 GB RAM, using the Microsoft Windows 7(64 bit) OS. A MATLAB toolbox, *i.e.*, YALMIP, is employed as the modeling tool in the numerical experiment (Löfberg, 2004). YALMIP is a modeling language for advanced modeling and solution of optimization problems, which is implemented as a free toolbox for MATLAB. As the solver, we use Gurobi 5.6.3, which is a state of the art optimization solver for linear programming.

Metro Line 6 is an east-west traffic corridor of Beijing, connecting the eastern suburban and urban center. Two stations in east part of Metro Line 6 are equipped with overtaking facilities, and the express/local mode discussed in this paper can work in this part. From east to west, the station set is **N** = {"Lucheng", "Dongxiayuan", "Haojiafu", "Beiyunhedong", "Beiyunhexi", "Yuntongmen", "Tongzhoubeiguan", "Wuzixueyuanlu", "Caofang", "Changying", "Huangqu", "Dalianpo"}, which are numbered from 1 to 12. In the experiment, we set the direction $1 \rightarrow 12$ as the upstream direction, and the rail links are marked by red numbers (see Fig. 10). Station 12 is the last major station that the express/local mode covers, and after station 12, there are another 16 stations, where only standard stop mode works.

As is shown, station 6 and 10 are equipped with overtaking facilities, *i.e.*, $\tilde{\mathbf{X}} = \{6, 10\}$. Besides the overtaking stations, there are another 4 major stations, *i.e.*, $\mathbf{X} = \{1, 4, 6, 8, 10, 12\}$. In Fig. 10, major stations are marked by solid dots.

Passenger flow in east part of Line 6 can be apparently divided into two types, namely, from suburban to center and within suburban, the ratio of which is about 3:1. The heterogenous passenger flow gives the motivation of introducing express/local mode to planners. In this paper, it is assumed that the passenger flow is steady during the time horizon ($H = 3$ h). Since the timetable is periodic, we only need to investigate the passenger flow λ_{pq} during one period, which is set to be $h_0 = 6$ min. Table 2 presents the value of λ_{pq} within h_0. Since we are only concerned with the upstream direction, the OD matrix is an upper triangular one.

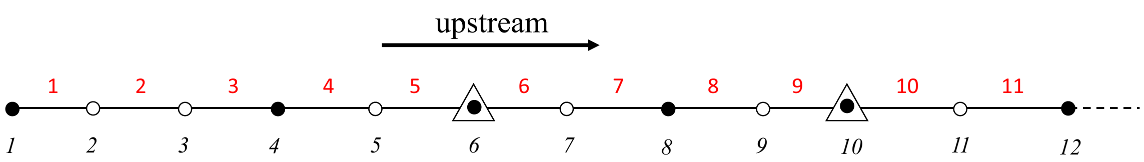

Fig. 10. East part of Beijing Metro Line 6.

Table 2
Passenger flow within h_0.

λ_{pq}	St.1	St.2	St.3	St.4	St.5	St.6	St.7	St.8	St.9	St.10	St.11	⩾St.12
St.1	–	5	5	15	5	30	5	30	5	25	15	400
St.2	–	–	5	20	5	15	5	5	5	15	5	300
St.3	–	–	–	5	5	10	5	5	5	20	5	200
St.4	–	–	–	–	5	30	10	20	5	25	10	350
St.5	–	–	–	–	–	5	5	15	5	15	5	200
St.6	–	–	–	–	–	–	5	20	5	20	10	300
St.7	–	–	–	–	–	–	–	10	5	10	10	150
St.8	–	–	–	–	–	–	–	–	5	20	10	250
St.9	–	–	–	–	–	–	–	–	–	15	5	100
St.10	–	–	–	–	–	–	–	–	–	–	5	100
St.11	–	–	–	–	–	–	–	–	–	–	–	100
St.12	–	–	–	–	–	–	–	–	–	–	–	–

The length of railway links and bounds of link running time are all listed in Table 3. Each train consists of 8 cars, and has a capacity of 1800 passengers. The average weight of passenger is about 65 kg, *i.e.,* $P = 65$ kg and the mass of an empty train is about 280,000 kg, *i.e.*, $G = 28{,}000$ kg. The values of other parameters are $d_0 = 30$ s,$d_1 = 150$ s,$h_s = 120$ s,$h_w = 45$ s,$h_r = 45$ s,$a = 1$ m/s^2, $b = 0.85$ m/s^2, and $c = 0.1$ m/s^2.

5.1. Comparison between standard mode and express/local mode

We first compare the operations between standard mode and express/local mode. In this part, we are particularly interested in the comparison of energy consumption when travel time is minimized.

If the standard stop mode is employed, in order to pursue the minimum travel time, link running times all attain their lower bounds, *i.e.*, $t_i^{std} = t_{1i}^L$. Besides, dwelling time at each station should be d_0 and the headway should be $h_{std} = h_0/2 = 3$ min. In this situation, it is easy to verify that formulations for passengers' total travel time and trains' traction energy consumption during h_0 are

$$T_{std} = \sum_{1 \leqslant p < q \leqslant n} \lambda_{pq} \left(\frac{h_0}{4} + \sum_{p \leqslant v \leqslant q-1} t_i^{std} + (q-p-1)d_0 \right),$$

$$W_{std} = 2GW^L(1,n) + \sum_{1 \leqslant p < q \leqslant n} \lambda_{pq} PW^L(p,q).$$

where $W^L(p,q)$ is defined by formula (16). For the east part of Beijing Metro Line 6, via substituting parameters, we obtain $T_{std} = 3.14 \times 10^6$ s and $W_{std} = 1.71 \times 10^9$ J.

We then investigate the situation when the express/local stop mode is employed (shown in Fig. 10). First, we need to obtain passenger numbers in trains according to the method in Appendix A. Table 4 presents the passenger numbers in a train when the train is about to depart from each station.

Second, input passenger number into objective functions. In this part, we are concerned with energy consumption when the minimum travel time is achieved, so the bi-objective model degenerates to single objective (travel time) model. Solving

Table 3
Data of link length and running time.

Link	$len(i)$	t_{1i}^L	t_{2i}^L	t_{1j}^X	t_{2j}^X
1	1200 m	90 s	105 s	230 s	245 s
2	1350 m	95 s	110 s		
3	950 m	80 s	95 s		
4	1600 m	110 s	125 s	190 s	210 s
5	1550 m	110 s	125 s		
6	1450 m	110 s	125 s	200 s	220 s
7	2550 m	170 s	185 s		
8	2100 m	150 s	165 s	180 s	210 s
9	1400 m	110 s	120 s		
10	1850 m	120 s	135 s	180 s	210 s
11	1250 m	90 s	105 s		

Table 4
Passenger number in train.

Station	Standard	Local	Express
1	270	277	263
2	458	653	
3	583	906	
4	790	1100	480
5	905	1330	
6	1040	579	1501
7	1115	729	
8	1205	818	1592
9	1245	898	
10	1215	818	1613
11	1225	838	

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the linear programming model, we obtain the following schedule (Table 5) with $h_{LX} = 200$ s, which leads to the minimum total travel time.

In the schedule of Table 5, the minimum total travel time is $T^*_{e/l} = 2.97 \times 10^6$ s, and the corresponding energy consumption is $W_{e/l} = 1.55 \times 10^9$ J. The time-distance diagram for minimum total travel time is shown in Fig. 11.

Compared with standard stop mode, total travel time is reduced by the express/local mode, *i.e.*,

$$\frac{T_{std} - T^*_{e/l}}{T^{std}} = \frac{3.14 - 2.97}{3.14} = 5.4\%.$$

The reduction of traction energy consumption is, *i.e.*,

$$\frac{W_{std} - W_{e/l}}{W_{std}} = \frac{1.71 - 1.55}{1.71} = 9.4\%.$$

Obviously, the reduction of energy consumption is much larger than travel time when the express/local stop mode is used.

Note that energy consumption and travel time are conflicting objectives, which indicates that the minimum travel time leads to higher energy consumption. Even so, the reduction of energy consumption is nearly 10%, which emphasizes the effectiveness of lowering energy consumption in express/local stop mode.

5.2. Pareto frontier of energy consumption and travel time

For objectives T_{total} and W_{total} in model (41), they are usually conflicting objectives. Qualitatively, reducing travel time usually requires raising running speed, which leads to higher energy consumption. In this section, we still take Beijing Metro Line 6 with express/local mode as an example to investigate the relationship between energy consumption and travel time. Note that the passenger flow and parameters are the same as those in Section 5.1.

The value of T_{total} is given in time unit, while the value of W_{total} is given in energy unit. For further quantitative research, these objectives must be normalized to a common scale. In this paper, we employ the following form of normalization,*i.e.*

$$T'_{total} = \frac{T_{total}}{T^*_{e/l}}, \quad W'_{total} = \frac{W_{total}}{W^*_{e/l}},$$

where $T^*_{e/l}$ is the minimum passengers' total travel time and $W^*_{e/l}$ is the minimum train traction energy consumption. We can obtain $T^*_{e/l}$ and $W^*_{e/l}$ by solving corresponding single objective of model (41), which is a linear programming problem, *i.e.*

Table 5
Schedule for minimum total travel time.

Link	t_i^L	t_j^X
1	90 s	230 s
2	95 s	
3	80 s	
4	110 s	190 s
5	110 s	
6	110 s	200 s
7	170 s	
8	150 s	180 s
9	110 s	
10	120 s	180 s
11	90 s	
	Link running time	
Station	d_i^L	d_j^X
2	30 s	–
3	30 s	–
4	30 s	30 s
5	30 s	–
6	150 s	30 s
7	30 s	–
8	30 s	30 s
9	30 s	–
10	120 s	30 s
11	30 s	–
12	30 s	30 s
	Station dwelling time	

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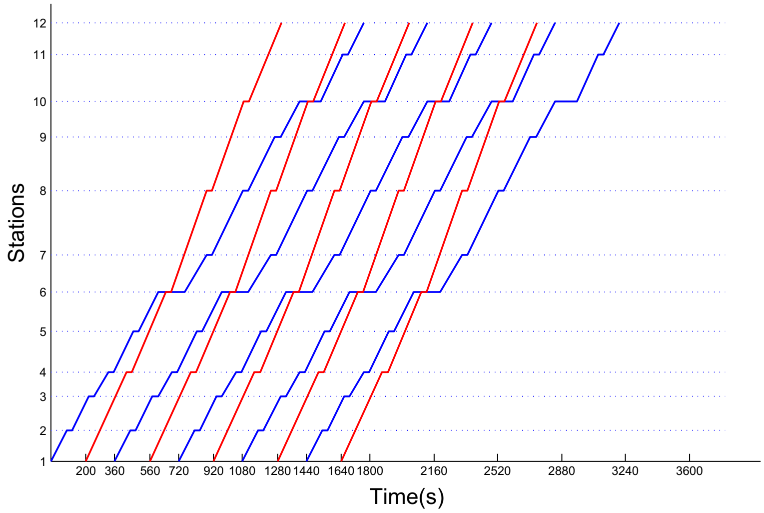

Fig. 11. Time-distance diagram for minimum travel time.

$$T^*_{e/l} = 2.97 \times 10^6 \text{ s}, \quad W^*_{e/l} = 1.35 \times 10^9 \text{ J}.$$

Obviously, $T'_{total} \geq 1$ and $W'_{total} \geq 1$, which can be regarded as the ratio of these objectives to their minimum values respectively. Define a new objective function for model(41), *i.e.,*

$$F_\alpha = \alpha T'_{total} + (1 - \alpha) W'_{total}. \tag{42}$$

As a result, F_α is referred to as a compromise objective to the bi-objective optimization model (41), which leads to a new single objective model

$$\begin{cases} \min & F_\alpha \\ s.t. & \text{constraints } (1)\text{–}(11). \end{cases} \tag{43}$$

Fig. 12 shows the frontier of (T_{total}, W_{total}) with α increasing from 0 to 1 by step 0.01, which is actually a Pareto frontier. Fig. 12 quantitatively illustrates the conflict between reducing travel time and lowering energy consumption, which implies that a tradeoff should be made. We find that when $\alpha = 0.75$, the travel time in express/local mode is

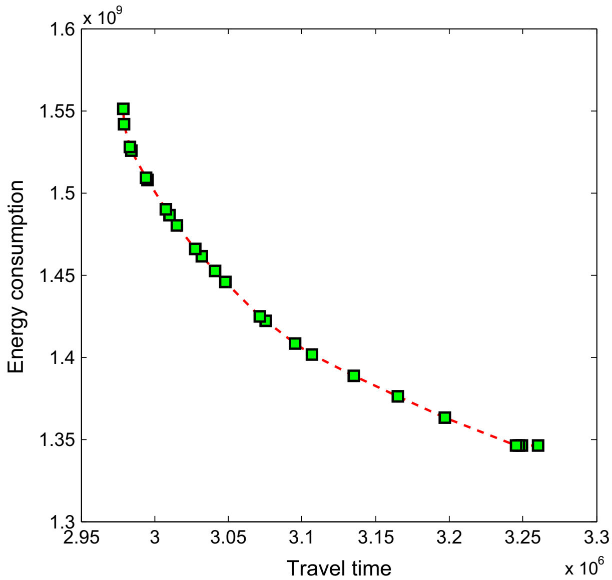

Fig. 12. Graph of (T_{total}, W_{total}).

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Table 6
A trade-off schedule.

Link	t_i^L	t_j^X
1	105 s	230 s
2	110 s	
3	80 s	
4	110 s	190 s
5	110 s	
6	125 s	200 s
7	170 s	
8	150 s	180 s
9	115 s	
10	120 s	180 s
11	105 s	
	Link running time	
Station	d_i^L	d_j^X
2	30 s	–
3	30 s	–
4	30 s	30 s
5	30 s	–
6	150 s	30 s
7	30 s	–
8	30 s	30 s
9	30 s	–
10	120 s	30 s
11	30 s	–
12	30 s	30 s
	Station dwelling time	

$T_{total} = 3.008 \times 10^6$ s and energy consumption is $W_{total} = 1.49 \times 10^9$ J. Compared with the results in standard mode, *i.e.*, $T_{std} = 3.14 \times 10^6$ s and $W_{std} = 1.71 \times 10^9$ J, we have

$$\frac{T_{std} - T_{total}}{T_{std}} = \frac{3.14 - 3.008}{3.14} = 4.2\%.$$

and

$$\frac{W_{std} - W_{total}}{W_{std}} = \frac{1.71 - 1.49}{1.71} = 12.9\%.$$

That is, the express/local mode with $\alpha = 0.75$ reduces total travel time and energy consumption by 4% and 12.9% respectively. The corresponding link running times and station dwelling times are listed in Table 6.

5.3. Computational time

At last, we investigate the complexity of the method used in this paper. Note that the objectives and constraints of model (41) are all linear functions of decision variables, which means that solving the optimization model is not time-consuming. The most time-consuming part of the method is fitting the minimum energy consumption functions by linear functions. For the example in Fig. 10, the fitting process is repeated 16 times, and the total computational time is about 4.1 s.

If we are only interested in minimizing total travel time, model (41) degenerates to a linear programming with single objective. Since it is assumed that the timetable is homogenous and cyclical, we only focus on the first pair of express/local trains. In this case, the computational time of Gurobi is less than 0.01 s, and the modeling time of Yalmip is about 0.58 s.

If we want to obtain the Pareto frontier (as in Fig. 12), the modeling process and solving process will be repeated 101 times, which means that the modeler (Yalmip) and the solver (Gurobi) will be called 101 times. In this case, the computational time is about 6.34 s.

6. Conclusions and future work

Due to the increasing environmental pressure, the issues on energy consumption in transport system attract more and more attention. In this paper, we investigated the relationship among energy consumption, travel time and timetable in a metro line with express/local mode. Based on the analysis of energy consumption in different OD cases, we developed a bi-objective linear programming model, which considered energy consumption and travel time simultaneously. Numerical experiments were performed on the Beijing Metro Line 6, in which express/local facilities are equipped. The results showed that compared with traditional standard stop mode, the express/local mode can lead to reductions both in energy consumption and travel time. In the case of minimum travel time, energy consumption was reduced by about 9.4% and travel time was reduced by 5.4%. Besides, it was found that there exists a Parito frontier between energy consumption and travel time.

It should be pointed out that in this paper, we only considered the case of a metro line. In future work, we should extend the model to a metro network. In a network, transfer between different lines must be considered, and therefore other solution methods should be developed to deal with the more complex passenger flow. Furthermore, in this paper, the ratio of express train and local train was fixed, *i.e.*, 1:1. A possible direction of future work is to investigate the energy consumption when this ratio depends on passenger flow. Besides, the passenger flow in this paper was assumed to be static, however, it is dynamic in practice. Hence, integrating dynamic passenger flow in the model is another research direction.

Acknowledgments

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Appendix A. route choice in express/local mode

With the introduction of express/local mode, passengers from station p to station q may have more routes instead of one. Specifically, some passengers may take a local train from p to q directly with no transfer, while some passengers may transfer once or twice at overtaking stations for a less travel time. In order to investigate total travel time, we must grasp the route choice of passengers, at least understanding it macroscopically. Meanwhile, diversified route choice complicates the calculation of passenger number in trains, which is a significant parameter in energy consumption.

Route choice in our problem is much simpler than that in general sense, since it only refers to a choice situation among two services in the same metro line, and the main influence factor is travel time. However, as is pointed out, route choice is always a tough but unavoidable problem in transportation research (Bekhor et al., 2006; Guo, 2008). Roughly, there are two related theories underlying the influence of route choice, namely behavioral science and economics, which often work together rather than separately. In express/local mode, route choice is mainly embodied in passengers' transfer to express trains at overtaking stations, namely, at which overtaking station the transfer happens and how many passengers will transfer to express trains. In the following, we will analyze route choice in our problem and then formulate it.

For simplicity, we set $1 \rightarrow n$ as the upstream direction, station p as the starting point and station q as the end point of a travel, where $p, q \in \mathbf{N}, p < q$. We first assume that transfer from a local train to an express train only happens at the first overtaking station after the starting point p. Take the travel $3 \rightarrow 12$ in Fig. 13 as an example, in which the transfer is assumed to happen at station 6, *i.e.*,route $3 \overset{local}{\rightarrow} 6 \overset{express}{\rightarrow} 12$. Since the route $3 \overset{local}{\rightarrow} 10 \overset{express}{\rightarrow} 12$ surely costs more time than the route $3 \overset{local}{\rightarrow} 6 \overset{express}{\rightarrow} 12$, it's unlikely that passengers who prefer a shorter travel time transfer to an express train at station 10 instead of at station 6. Besides, we also assume that transfer from an express train to a local train only happens at the last overtaking station before the end point q. For example, the travel $2 \rightarrow 9$ in Fig. 13, in which the transfer is assumed to happen at station 6, *i.e.*,route $2 \overset{local}{\rightarrow} 6 \overset{express}{\rightarrow} 9$.

Considering the bi-directional nature of metro lines, it is possible that passengers first take an upstream express train to 10 and then transfer to a downstream local train to 9, *i.e.*,$2 \overset{express}{\rightarrow} 10 \overset{local}{\rightarrow} 9$. However, bi-directional routes like $2 \overset{express}{\rightarrow} 10 \overset{local}{\rightarrow} 9$ are not considered in the situation of this paper. Note that it is assumed that the service frequency of the metro line is moderate, and express trains and local trains depart alternatively. Adding the travel time of route $10 \overset{local}{\rightarrow} 9$, it is very likely that bi-directional route $2 \overset{express}{\rightarrow} 10 \overset{local}{\rightarrow} 9$ costs more time than the route $2 \overset{local}{\rightarrow} 6 \overset{express}{\rightarrow} 9$. Moreover, compared with the latter, it is much more difficult to calculate the travel time of route $2 \overset{express}{\rightarrow} 10 \overset{local}{\rightarrow} 9$, since we have to know the timetable of the downstream direction. As a result, we ignore the bi-directional nature of metro lines in this paper.

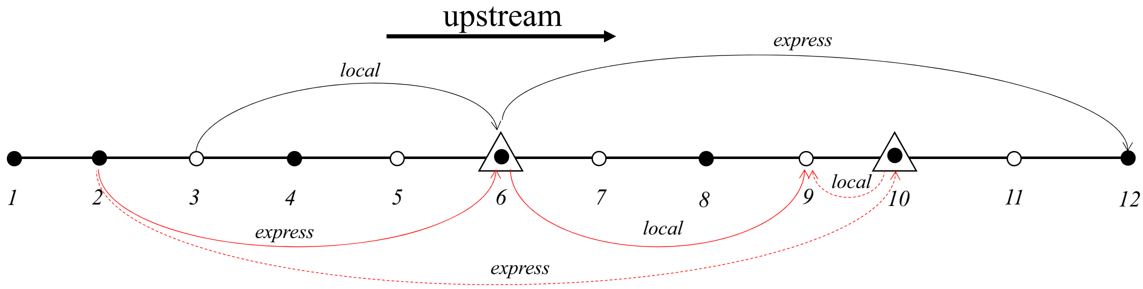

Fig. 13. Illustration for transferring analysis.

Next, we will discuss the number of passengers transferring from a local train to an express train at overtaking stations. This transfer behavior happens when an express train can provide a service of less travel time. It has been pointed out that human behavior is inherently probabilistic, and some researchers have used the random utility functions (Manski, 1977; Guo, 2008) to model the motivation of route choice. In this method, for route p_i, its utility function $U(p_i)$ is often calculated as the sum of different terms, such as travel time and ticket cost, *i.e.*,

$$U(p_i) = \sum_k \alpha_k \cdot Term_{i,k},$$

and the choice probability of route p_{i0} follows

$$Pr(p_{i0}) = \frac{e^{-\theta \cdot U(p_{i0})}}{\sum_i e^{-\theta \cdot U(p_i)}},$$

where θ is a parameter that needs to be predetermined.

Generally, the choice probability should be a balance between theoretical validity and computational convenience. In the case of express/local mode, the main factor that influences choice probability is travel time. In our paper, however, travel time of any route is not predetermined, since link running times and station dwelling times are all decision variables. As a result, if the choice probability involves travel time, the subsequent computation will be very complicated. Thus, it is not suitable to use the method of utility function.

As stated above, passengers in local trains have a motivation to transfer to express trains at overtaking stations. However, transfer behavior itself is troublesome. Considering the frequency of metro lines in this paper, if the transfer will not save enough time, some passengers prefer remaining in local trains. Because an express train skips minor stations, which means that it saves dwelling time at the skipped stations and corresponding acceleration/deceleration time, it has a higher average speed than a local train. Obviously, the more stations are skipped, the more travel time is saved by an express train. This means that the motivation of a passenger's transfer is greater if his trip covers more station. With this in mind, we propose a brief formula to roughly grasp the transfer probability, which is mainly determined by the locations of starting point p and end point q. To be precise, for passenger getting on a local train at station p, at the first overtaking station after p, if transfer to an express train can reduce the travel time, he/she will transfer with probability K_{pq}, where K_{pq} is defined by

$$K_{pq} = (q - p)/n, \quad 1 \leqslant p < q \leqslant n. \tag{44}$$

Note that for some p and $q, K_{pq} = 0$. This is because in some cases, there is no need to transfer to an express train from a local train, for example from station 2 to station 5, or from station 3 to station 7 (Fig. 13). It is easy to verify that only (i) when there are more than one overtaking stations between p and q, and (ii) when q is a major station and there is only one overtaking station is between p and q, transfer from a local train to an express train is time-saving. In average, there are K_{pq} percent of passengers will transfer from a local train to an express train. At overtaking station p, we still assume that K_{pq} of passengers will take an express train, where K_{pq} is calculated by formula (44).

We want to emphasize that our research is not limited to the transfer probability formula (44), and it can be based on an arbitrary route choice module, which satisfies: (i) the probability is independent with link running time and station dwelling time, (ii) the probability increases if more stations are skipped. Actually, formula (44) is one of the simplest ones that roughly describe the main characteristics of transfer in express/local mode. The more complicated and more exact formula can be computed based on passenger inquiries and historical data fittings. However, this is out of the scope of this paper.

Appendix B. calculation of energy consumption

More and more subway systems have employed new technologies to improve their energy management, including reducing traction energy consumption via improving driving regimes. According to the theory of optimal control, running on straight rail links, a train's optimal driving strategy contains the following four phases in turn: maximum acceleration, cruising, coasting and maximum braking (Howlett and Pudney, 1995; Albrecht, 2008). Since it isn't a paper on detailed energy calculation, we employ the simplest model to describe the problem. To be more precise, maximum acceleration, resistance per unit mass and maximum braking are assumed constants in this paper. Then, given distance S and running time T, the speed profile of optimal driving strategy can be roughly presented by the curve $OACBT$ in Fig. 14, where OA is maximum acceleration phase, AC is the cruising phase, CB is the coasting phase, BT is the maximum braking phase. In Fig. 14, s_i represents the distance that a train travels under corresponding phase. Obviously, $s_1 + s_2 + s_3 + s_4 = S$.

As mentioned above, a is the maximum acceleration, c is the resistance of motion per unit mass and b is the maximum braking. On per unit mass, during the acceleration phase, the traction F_1 can be calculated by the Newton's second law, *i.e.*,

$$F_1 - c = a.$$

During the cruising phase, the traction F_2 is used to counteract the resistance of motion, *i.e.*

$$F_2 = c.$$

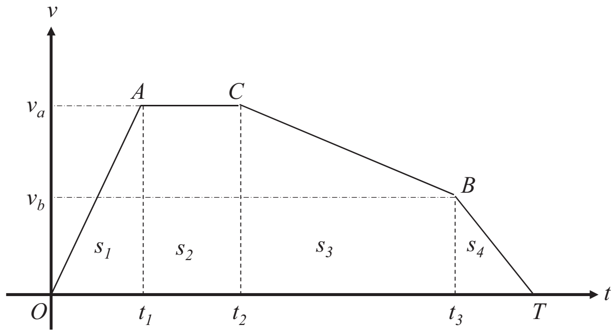

Fig. 14. Speed profile for minimum traction energy consumption.

During the coasting phase, trains decelerate only by resistance; while during the braking phase, trains decelerate by both resistance and braking, and the composition of these two forces is the maximum deceleration b. During these two phases, the engine is stopped, which indicates that the traction is nil, *i.e.*,

$$F_3 = F_4 = 0.$$

Thus, energy is mainly consumed in the acceleration and cruising phases, *i.e.*,

$$E = E_1 + E_2 = F_1 s_1 + F_2 s_2.$$

It is easy to verify that

$$s_1 = \frac{1}{2}at_1^2, \quad s_2 = at_1(t_2 - t_1), \quad s_3 = \frac{1}{2}(at_1 + b(T - t_3))(t_3 - t_2), \quad s_4 = \frac{1}{2}b(T - t_3)^2.$$

Then, the energy consumption can be formulated as

$$E = \frac{1}{2}at_1^2(a + c) + act_1(t_2 - t_1). \tag{45}$$

Given distance S and link running time T, the minimum energy consumption can be obtained by the following constrained optimization problem.

$$\begin{cases} \min\limits_{t_1,t_2,t_3} & E(S,T) = \frac{1}{2}at_1^2(a+c) + act_1(t_2 - t_1) \\ \text{s.t.} & v_a = at_1 \\ & v_b = b(T - t_3) \\ & v_a - v_b = c(t_3 - t_2) \\ & s_1 + s_2 + s_3 + s_4 = S \\ & 0 < t_1 \leqslant t_2 \leqslant t_3 \leqslant T. \end{cases} \tag{46}$$

where t_1, t_2 and t_3 are the switching points.

Under the assumption, the optimal control problem has been simplified to a constrained optimization problem (46), the exact solution of which can be theoretically obtained. However, since both the constraints and objective function are quad-

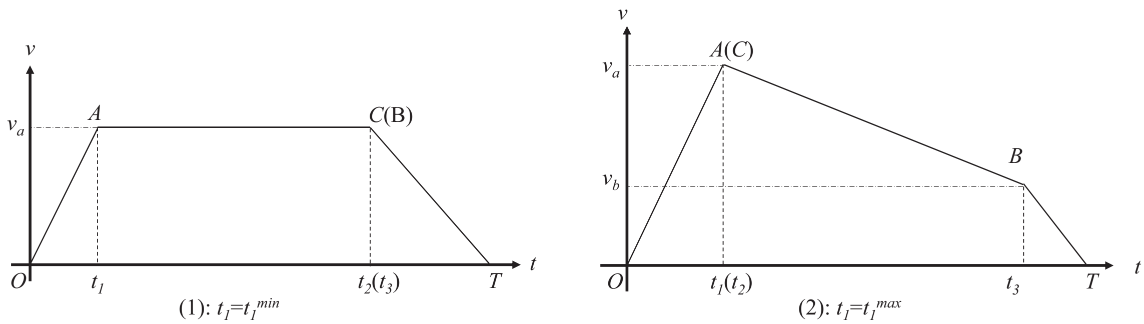

Fig. 15. Minimum and maximum acceleration time.

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ratic functions of t_i, to obtain the analytical solution is a difficult task, even for the powerful symbolic computation software Wolfram Mathematica 9.0.

According to constraints of problem (46), we find that if t_1 is given, t_2 and t_3 can be unique determined by solving a quadratic equation. In fact, the minimum and maximum value of t_1 can be determined if distance S and link running time T is given, see Fig. 15. In the situation of $t_1 = t_1^{min}$, since the maximum(cruising) speed v_a is low, in order to satisfy the constraint of link running time, the coasting phase is omitted, which indicates $t_2 = t_3$ (see Fig. 15(1)). If $t_1 < t_1^{min}$, the running time will be greater than the given value T. In the situation of $t_1 = t_1^{max}$, since the maximum(cruising) speed v_a is high, in order to satisfy the constraint of link running time, the cruising phase is omitted, which indicates $t_1 = t_2$. If $t_1 > t_1^{max}$, the running time will be less than the given value T. In short, the optimal value of t_1 is located in the interval $[t_1^{min}, t_1^{max}]$, and can be approximated via traversing $[t_1^{min}, t_1^{max}]$ by a small enough step δt.

Based on above analysis, we design the following iterative algorithm. For simplicity, set $E_0(S,T) = \min_{t_1,t_2,t_3} E(S,T)$.

Algortithm 1. *Given S, T and δt, the algorithm for calculating the minimum energy consumption is summarized as follows.*

Step 1: Calculate t_1^{min} and t_1^{max}. Set $t_1 = t_1^{min}$ and $E_0(S,T) = +\infty$.
Step 2: If $t_1 < t_1^{max}$, go to Step 3; otherwise, terminate the algorithm.
Step 3: Based on t_1, calculate t_2 and t_3. Then calculate the value of $E(S,T)$ according to formula (45).
Step 4: If $E_0(S,T) < E(S,T)$, set $E_0(S,T) = E(S,T)$. Set $t_1 = t_1 + \delta t$. Go to Step 2.

The final value of $E_0(S,T)$ is the minimum energy consumption. The numerical experiment on MATLAB on a personal computer shows that when $S = 1400$ m, $T = 95$ s and $\delta t = 0.01$ s, the computational time is less than 0.001 s. Considering that the maximum speed of trains is usually less than 30 m/s, the precision and computational time of the algorithm are acceptable.

Mathematically, fixing distance S, the minimum energy consumption $E_0(S,T)$ is a function of link running time T. Algorithm 1 gives a numerical method to calculate $E_0(S,T)$, and we can store the data of $(T, E_0(S,T))$ in computers. However, when function $E_0(S,T)$ is involved in an optimization model, it is inconvenient to handle if there is no explicit expression of $E_0(S,T)$. On the other hand, in an optimization model, the upper and lower bounds of T is usually given, *i.e.*, $T \in [t_{lower}, t_{upper}]$. We find that if the gap between t_{upper} and t_{lower} is small, the graph of $(T, E_0(S,T))$ can be well fit by a linear function.

$$E_0(S,T) \doteq E'(S,T) = \alpha + \beta \cdot T, T \in [t_{lower}, t_{upper}] \tag{47}$$

Assume that $a = 1$ m/s^2, $c = 0.1$ m/s^2, $b = 0.85$ m/s^2, and $S = 1400$ m, $T \in [95$ s, 110 s$]$. Fit the graph of $(T, E_0(S,T))$ in the interval $[95, 110]$, then we have

$$E_0(S,T) \doteq E'(S,T) = 618.75 - 3.92 \cdot T, T \in [95, 110].$$

This approximation method has a quite small relative error. Fig. 16 illustrates the functions $E_0(S,T)$ and $E'(S,T)$, in which the maximum relative error is about 2%, *i.e.*

$$\epsilon = \max_{t \in [95,110]} \frac{|E_0(S,T) - E'(S,T)|}{E_0(S,T)} = 2.03\%.$$

Moreover, as S and T increase, the maximum relative error decreases rapidly. For example, when $S = 2300$ m and $T \in [135$ s, 150 s$]$, we have $\epsilon = 0.73\%$; when $S = 3000$ m and $T \in [195$ s, 210 s$]$, we have $\epsilon = 0.24\%$.

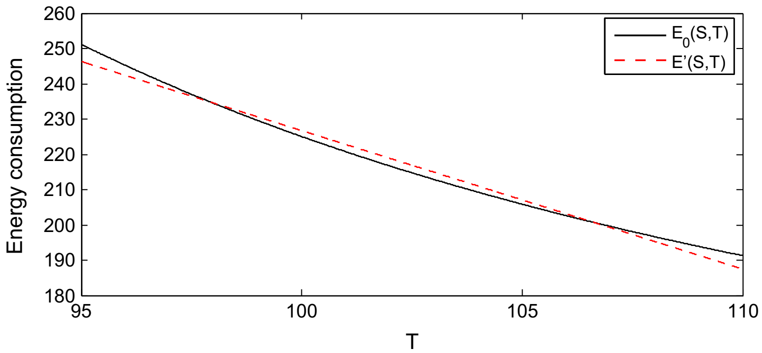

Fig. 16. Functions $E_0(S,T)$ and $E'(S,T)$.

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